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Three essays on risk and uncertainty in agriculture

Nicholas David Paulson
Iowa State University

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Three essays on risk and uncertainty in agriculture

by

Nicholas David Paulson

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Bruce A. Babcock, Co-major Professor
Dermot J. Hayes, Co-major Professor
David A. Hennessy
Sergio H. Lence
Alicia L. Carriquiry
Chad E. Hart

Iowa State University

Ames, Iowa

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ABSTRACT

The general theme of this dissertation is risk and uncertainty in agriculture, with each chapter addressing a specific topic related to agricultural risk and uncertainty. Chapter 2 examines the effects of production uncertainty on the types of contract structures used in specialty grain markets. A theoretical model of a contractual relationship between a monopsonistic processor and risk-neutral producers is presented. Two common contract structures, and their resulting effects on the sharing of production risk between buyer and seller, are compared. The spatial structure of yields and farm-level yield volatility are shown to have significant impacts on the processor's preferred choice of contract structure and expected profits of both the processor and farmers in the resulting equilibrium. Chapter 3 provides a critical look at a classic definition regarding the relationship between input use and risk, and attempts to reconcile an apparent paradox in the production literature. Experimental corn yield response data is used to estimate a stochastic production relationship between applied fertilizer, soil nutrient availability, and corn output. Optimal fertilizer application rates for risk-averse and risk-neutral producers are found using numerical methods. In addition to the empirical analysis, primary data collected through a farmer survey instrument, designed to elicit information from farmers regarding their risk attitudes and subjective beliefs regarding the relationship between risk and fertilizer use, is presented and compared with the results of the empirical analysis. Chapter 4 turns to the opportunities for managing weather risk using weather derivative markets. Developing regions are areas in which weather based risk management tools show significant potential. However, the success and long-term viability of insurance programs depends heavily on the availability of accurate and reliable historical data. The lack of this type of historical data for developing

regions is one of the largest obstacles to insurance program development in these regions. A framework which utilizes statistical methods to estimate unbiased rainfall histories from sparse data is developed. To validate the methodology's usefulness, a drought insurance example is presented using a rich data set of historical rainfall at weather stations across the state of Iowa.

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CHAPTER 1. GENERAL INTRODUCTION

Introduction

It would be difficult to imagine an industry where risk and uncertainty are more important than in agriculture. Throughout the supply chain, agents are faced with sources of uncertainty which may be exogenous, endogenous, or both. At the farm level, livestock and crop producers are forced to make input choices and asset allocation decisions in a complex environment of volatile prices, perishable outputs, production lags, seasonality effects, and increasing concentration at all levels of the supply chain. Additionally, producers are constantly at the mercy of extreme weather variability. Similarly, downstream participants must efficiently coordinate with producers to ensure a steady supply of inputs for further processing and eventual sale to end users. Therefore, the development of carefully planned and well defined risk management strategies is crucial to the continued success of all agents in the agriculture industry. As agriculture continues to evolve, new sources of risk continue to arise presenting new challenges for both buyers and sellers of agricultural commodities.

The use of contracts as a coordination mechanism in agriculture is an area that has seen considerable recent growth. Increasing real incomes and consumers' subsequent demand for more highly differentiated products is often cited as one of the main forces in this area. While contracts can provide a coordination mechanism to improve efficiency in emerging marketplaces, they also introduce new challenges. Contracts must be structured to efficiently allocate the added value, risk, and decision making rights between the buyer and seller. Additionally, the incentives of buyers and sellers are often misaligned, adding yet another layer of complexity to contract design. Buyers would like to eliminate production

and price risk while procuring specialty products at the lowest possible price. On the other hand, the goal of producers is to eliminate production and price risk while trying to sell their production at the highest possible price. For a long-run equilibrium to emerge, contract structures must evolve to reflect an efficient tradeoff between the added-value created by specialty products and the levels of price and production risk borne by buyers and sellers.

Thus far, the academic research has focused more on contracting within the livestock industry. The co-existence of spot and contract markets, and the resulting effect on the forces of competition, has also garnered a significant amount of attention. Research on contract structures in crop production has been given much less attention despite the increasing use of production contracts in the growing markets for specialty crops. Further research on contract structures would provide additional insight on the implications of contract structure on the sharing of price and production risk between buyers and sellers of agricultural commodities. This topic is the focus of one of the following chapters.

Beyond marketing alternatives, one of the most fundamental risk management strategies available to producers is that of the choice of input mixes. A producer's optimal input mix will depend on many factors, including his risk preferences and attitudes. Producers want to avoid excessive use of inputs to control production costs while also ensuring that the input mix does not limit production when prices and conditions in the production environment are such that profits can be made. The relationship between risk and input use has been extensively studied in the academic literature. In the specific case of fertilizer use, there exists somewhat of a paradox. Empirical evidence implies that the variability of production is increasing in the amount of fertilizer applied by the producer.

Therefore risk-averse agents are predicted to use lower amounts of fertilizer than risk-neutral

farmers. However, there is also a significant amount of empirical evidence that fertilizers are over-applied by farmers as an act of self-protection. A clear reconciliation of the seemingly opposing viewpoints would provide a contribution to the production literature. A chapter of this dissertation is devoted to this very topic.

While many sources of risk and uncertainty are at least partially endogenous, weather variability is most certainly out of the control of participants in the agriculture industry. Mother Nature can turn an otherwise bumper crop into a total failure with one hard rain or hailstorm regardless of the input mix chosen by the producer. Similarly, excessively hot or cold weather can have dramatically negative impacts on livestock productivity in even the most efficient operations. Recent growth in the weather derivative industry has created new opportunities for weather risk management in agriculture. Options and insurance contracts based on weather can provide income transfers when weather conditions adversely affect farm profits. While the demand for insurance based on weather events may be limited in areas with well developed crop and livestock insurance programs, such as the United States and Canada, a large body of literature is devoted to exploring their potential for developing regions. However, the lack of reliable historical data needed for proper program development is a major obstacle to weather derivative growth in developing regions. Despite the challenges, the potential for weather based products to enhance incomes and provide increased stability in developing regions warrants further research. In this dissertation, I provide a potential solution to the problem of unavailable historical weather data for developing areas and illustrate an implementation of the framework.

Organization of the Dissertation

This dissertation is organized into five chapters. The current chapter includes a general introduction to the chapters that follow and, in this section, provides an outline for the document's organization. While the general theme of this dissertation is risk and uncertainty in agriculture, each chapter is meant to stand alone by addressing a specific component of the issues discussed in the previous section.

I begin in Chapter 2 by taking a look at the effects of production uncertainty on the types of contract structures used in specialty grain markets. While contracts themselves can reduce risk by matching buyers and sellers in specialized markets, the structure of a contract determines how value, risk, and decision-making rights are allocated between the buyer and seller. A theoretical model of a contractual relationship between a monopsonistic processor and risk-neutral producers is presented. Two common contract structures are compared: acreage contracts and bushel contracts. Acreage contracts place a larger share of production risk on the buyer, while bushel contracts shift a greater portion of this risk to the producer. The effects of the spatial correlation of yields across multiple producers, and the volatility of yields at the farm level are shown to have a significant impact on the processor's preferred choice of contract structure and expected profits in the resulting equilibrium.

Chapter 3 provides a critical look at a classic definition of the relationship between input use and risk and attempts to reconcile an apparent paradox in the production literature. Specifically, I examine how input decisions may affect the level of production variability a producer faces (risk endogeneity). Experimental corn yield response data is used to estimate a stochastic production relationship between applied fertilizer (the farmer's choice variable), soil nutrient availability (a stochastic function of applied fertilizer), and corn output (a

stochastic function of available soil nutrients). Optimal fertilizer application rates for risk-averse and risk-neutral producers are found using numerical methods. The optimal rates are compared for agents with differing risk preferences. In addition to the empirical analysis, primary data collected through a farmer survey instrument is presented. The survey was designed to elicit information from farmers regarding their risk attitudes and their subjective beliefs regarding the relationship between risk and nitrogen fertilizer use. The survey results are compared and contrasted with those obtained from the empirical analysis.

Chapter 4 turns to the challenges created by weather variability in agriculture and the opportunities that are arising for managing weather risk. Specifically, the relatively new but increasingly popular market for weather derivatives is examined. While a wide variety of well developed insurance programs are available in the United States and Canada, agricultural insurance programs are not as widely available in other areas. Developing regions are areas in which weather based risk management tools show significant potential for further development. However, the success and long-term viability of insurance programs depends heavily on the availability of accurate and reliable historical data. The lack of this type of historical data for developing regions is one of the largest obstacles to insurance program development in these regions. The focus of Chapter 4 is on the development of a framework which utilizes a statistical method to estimate unbiased rainfall histories from sparse data. Specifically, a Bayesian spatial kriging model is used in conjunction with Markov Chain Monte Carlo methods to estimate historical rainfall for out-of-sample locations. These estimated rainfall histories can be used in place of actual historical data to develop an actuarially sound insurance program based on weather. To

validate the methodology's usefulness, a drought insurance example is presented using a rich data set of historical rainfall at weather stations across the state of Iowa.

Finally in Chapter 5, I provide a summary and some general conclusions.

CHAPTER 2. A COMPARISON OF ACREAGE AND BUSHEL CONTRACTS IN SPECIALTY GRAIN MARKETS

A paper to be submitted to the *American Journal of Agricultural Economics*

Nicholas D. Paulson

Abstract

The increase in vertical integration in agriculture has been motivated by many factors including the evolving demand of consumers as well as factors specific to agricultural markets (i.e. production and price uncertainty and farm policy). The literature on agricultural contracts has focused more on contracting in the livestock sector relative to crop production under contract, most likely due to the fact that contracting in livestock production has been historically more prevalent. However, crop production under contract has also realized extensive growth, especially in the markets for crops with specialty traits. This paper provides a theoretical model of a contracting relationship between a risk-neutral monopsonistic processor of a specialty crop and risk-neutral producers. The processor induces farmers to accept production contracts, based on acreage or total bushels, to grow the specialty crop by offering a premium above the commodity price for contracted production. It is commonly assumed in the literature that acreage contracts will be the preferred structure due to producers being relatively more risk averse than processors. However, this paper presents a market environment in which the opposite result is found. It is shown that a bushel contract exists which Pareto dominates the optimal acreage contract, although that bushel contract may not be the processor's optimal bushel contract, and that both the

processor and producers will be able to achieve higher expected profits by using a bushel contract structure. While explicit analytic solutions for the model do not exist, a numerical example is provided to illustrate effects of farm level yield volatility and the spatial correlation of farm yields on expected profits, premium levels, and farmer participation for the acreage and bushel contract equilibriums.

Introduction

The level of vertical integration in agricultural markets has seen considerable growth over the last decade. Authors have outlined many motivations for this phenomenon including supply-chain organization (Tsoulouhas and Vukina 1999), more discriminating consumers (Barkema 1993), more efficient relationships between buyers and sellers (Drabenstott 1993), information asymmetries (Hennessy 1996), quality control (Hueth and Ligon 1999; Hennessy and Lawrence 1999), procurement considerations specific to the dynamics of agricultural decision making (NASS 2003; Sexton and Zhang 1996), declining commodity prices (Fulton, Pritchett, and Pederson 2003), and the decoupling of farm support outlined in the 1996 farm bill (Coaldrake and Sonka 1993).

While a large proportion of grains and oilseeds are produced under marketing contracts in the United States (NASS 2003), production contracts have been relatively more prevalent in livestock (Goodhue 2000; Lawrence et al. 1997; Johnson and Foster 1994) and specialty grains markets (Ginder et al. 2000; Good, Bender, and Hill 2000; Fulton, Pritchett, and Pederson 2003; Sykuta and Parcell 2003). Specialty crop markets are generally smaller and more centralized than those for commodity crops. Risk management options for specialty grains are also imperfect as crop insurance, and futures and options markets are

generally only available for commodity crops. These characteristics make production and procurement in spot markets riskier for farmers and processors, respectively. The risk associated with spot market production and procurement is one of the main reasons given for producers entering into specialty grain contracts in Indiana (Fulton, Pritchett, and Pederson 2003).

Moreover, specialty crops are associated with higher production costs than commodity crops. Higher costs are attributed to factors such as increased labor intensity, storage issues stemming from segregation and identity preservation requirements, and specific or additional input requirements and field operations (Fulton, Pritchett, and Pederson 2003; Ginder et al. 2000). Production of specialty crops may also require the use of specific assets (Lajili et al. 1997; Sporleder 1992), which may be associated with a higher level of equity financing relative to less specific assets (Williamson 1996). Thus, processors must properly structure contract terms to reflect the additional costs and risks associated with the production of specialty crops to induce farmer acceptance of production contracts.

While considerable attention has been given to the effects of contract structures in the livestock industry, less attention has been giving to production contracts for crop production. This paper presents a theoretical model of a monopsonistic processor who procures crop production under contract from a set of producers. The processor sets the structure and terms of the contract as well as the number of producers to whom the contract is offered to maximize expected profits subject to a capacity constraint and producers' decision rules. Risk-neutral producers who are offered the contract then decide to either accept the contract or produce a commodity crop for sale on the commodity market to maximize expected profits. Producer heterogeneity is introduced with respect to production costs for the

specialty crop. While the situation is described as one where the processor contracts with producers to grow a specialty crop, the contracting relationship in the model could easily be generalized to other markets.

The contributions of this paper are two-fold. First, the model compares the equilibrium outcomes when contracts are based on acreage (acreage contracts) to when contracts are based on a specified production level (bushel contracts). Authors generally assume that acreage (bushel) contracts will be preferred by producers (processors), because they shift production risk from the producer (processor) to the processor (producer) (Sykuta and Parcell 2003; Lajili et al. 1997). However, the set of assumptions which define the market environment in this analysis lead to a different result. It is shown that under certain conditions, there exists a bushel contract that Pareto dominates the optimal acreage contract. Expected profits for both the processor and producers may be greater when the bushel contract structure is implemented rather than an acreage contract. However, that bushel contract may not be optimal in that it is not the processor's expected profit maximizing bushel contract.

Second, the model recognizes that the processor's total procurement in any period is the sum of random yield realizations on all contracted acres. The spatial correlation structure of yields will affect the processor's ability (or inability) to pool farm level production risk over a large number of contracted producers. This is expected to have an impact on the processor's preferred choice of contract structure (acreage vs. bushel contract). A numerical calibration to the model provides some insight into how the spatial correlation of producer yields may affect the processor's choice of contract type and terms. Equilibrium outcomes with respect to expected processor and producer profits are compared for a range of

parameterizations, finding that the processor will, in general, be able to achieve greater expected profits using bushel contracts. Expected producer profits are also shown to be greater under bushel contracts in all scenarios analyzed.

The assumption of a capacity constraint, producer risk-neutrality, and the effects of an *ex post* spot market for the specialty crop under bushel contracts are the main drivers of this result. The relative advantage of bushel contracts to acreage contracts is shown to increase as the level of correlation between farm level yields is increased. Intuitively, as yield risk becomes increasingly systemic in nature, the processor's benefits of placing a larger share of production risk on the producers increase. This result confirms that the choice of contract structure will hinge heavily on the poolability of production risk for specialty crops.

The next two sections provide a brief review of the literature on production contracts in agriculture and a summary of recent survey results from specialty grains markets in the Midwest, respectively. Then the contracting model is presented followed by a section providing some analytical results. A numerical calibration to the theoretical model is then provided with results for a range of parameterizations. The final section concludes and discusses possible extensions to the model and directions for further research.

Literature Review

Considerable attention has been given to the effects of contract structure on the sharing of value and risk in agricultural markets. Goodhue (2000) used an agency theory approach to model production contracts in the broiler industry, finding that contracts outlining relative compensation schemes and strict input control by the processor were optimal responses to grower heterogeneity and risk aversion. Weleschuk and Kerr (1995)

used a transactions cost approach to examine contracts for specialty crops in Canada, finding market power on the part of buyers led to reduced competition with respect to the compensation terms of contracts. Goodhue and Hoffman (2006) discuss the technical aspects of contracts, referred to as “boilerplate” in the contracting industry, that are often ignored in theoretical studies but play large roles in actual settings with regards to contract enforcement and liability issues.

Empirical approaches include those of Purcell and Hudson (2003) concerning vertical alliances in the beef industry, and Fraser’s (2005) examination of contracts in the Australian wine grape industry. Purcell and Hudson (2003) use a simulation model of cattle producers, cooperative feedlots, and beef packers to examine the effects of different compensation structures on risk sharing within a vertical beef alliance. Fraser (2005) applies regression analysis to actual contract data to identify the effects of grower and regional characteristics on contract structures. Fraser’s results are consistent with Sykuta and Cook’s (2001) assertion that producer characteristics have a limited effect in contract design, implying buyer characteristics will more often determine the specific contract terms.

Other authors have used experimental methods to elicit producer preferences for marketing contract attributes in both livestock (Roe, Sporleder, and Belleville 2004) and crop (Lajili et al. 1997) production. Lajili et al. (1997), using survey data, related the levels of asset specificity, production uncertainty, and producer risk aversion to the preferred levels of vertical integration with respect to the sharing of production risk and costs. Roe, Sporleder, and Belleville (2004) used an experimental survey design for marketing contracts in the hog industry, finding that producers strongly preferred contracts offered by cooperative firms, validating a hypothesis by Sykuta and Cook (2001) regarding institutional considerations in

agricultural contracting. Wu and Roe (2007) is another example of an experimental approach which illustrates the importance of third-party contract enforcement in contractual relationships when there is buyer concentration (market power).

Considerable attention has also been paid to the interaction between and the co-existence of contract and spot markets in agriculture. Xia and Sexton (2004) present a model of cattle production where buyer concentration leads to reduced *ex post* spot market (price) competition when contract premiums are based on cash market prices. Zhang and Sexton (2000) use a spatial model with high transportation costs to show how processors can use exclusive contracts to create captive supplies to gain monopsony power and reduce price competition on the spot market. However, Carriquiry and Babcock's (2004) model of oligopsony in specialty grain markets shows that if the contract premium is based on a fixed cash market price there will be increased competition on the *ex post* spot market. The recent work by Wang and Jaenicke (2006) uses a principal-agent framework within a market equilibrium model to show that the introduction of contract markets may cause spot prices and spot price volatility to rise or fall depending on the relative sizes of the contract and spot markets, and the compensation structures outlined in the contracts.

While there have been a number of excellent studies into the effects of contract structures on market equilibriums, authors have mainly focused on livestock markets and the compensation schemes in agricultural contracts. This is most likely due to the importance of price uncertainty relative to production uncertainty in the livestock sector. However, production uncertainty plays a major role in crop production. Therefore, one would expect production uncertainty to play a crucial role in shaping the contract structures in specialty grains markets. This is the main focus of this paper.

Survey Data

In addition to the theoretical and empirical studies on agricultural production contracts, there also exists a collection of recent survey data on specialty grain markets in the Midwest. The survey data reported in Good, Bender, and Hill (2000) for Illinois specialty grain handlers shows that the vast majority of specialty corn (waxy, high oil, white, yellow food grade) and soybeans (tofu, non-GMO) in Illinois are produced and procured under contractual arrangements. Good, Bender, and Hill (2000) report that many firms act as intermediaries between producers and processors by forming contractual relations with both parties. Moreover, the intermediate firms, comprised mainly of country elevators, contract with producers based on acreage but contract with processors based on bushels. This implies that the intermediate firms may be able to pool production risk across producers.

Sykuta and Parcell (2003) provide a survey on contract structures offered by DuPont in their specialty soybean programs. While the contract premiums are generally based on the total bushels delivered, the actual contracts are based on acreage, shifting a portion of the production risk to the processor. Acreage contracts differ from bushel contracts in that the producer does not have to make up yield shortages in poor years or sell surpluses at a potentially lower price on the spot market in good years. The production risk shifted to buyers under acreage contracts creates variability with respect to the amount of premium paid out to contracted producers, as well as serious implications for capacity considerations for processors. Sykuta and Parcell (2003) note that buyers may be able to reduce production risk by pooling across large areas (attempting to eliminate some systemic yield risk), or offer contracts in selective areas or to specific producers with low historical yield variability.

The survey data from over 2,800 producers in Ginder et al. (2000) reports that the majority (60%) of specialty corn contracts in Iowa are structured to pay a premium over a reference price, referred to as “market price plus a premium”. The reference price can either be a fixed price or pegged to a specified cash market price, such as a local spot market or a futures price. The surveys conducted by Good, Bender, and Hill (2000) and Fulton, Pritchett, and Pederson (2003) imply the same type of compensation structures for specialty grain contracts in Illinois and Indiana, respectively. Sykuta and Parcell’s (2003) survey also reported the use of a premium above a market price in DuPont’s specialty soybean programs.

The premiums are motivated by higher production costs, segregation and identity preservation, possible yield drags, and documentation and certifications costs (Ginder et al. 2000). The magnitude of these additional costs will depend on the characteristics of the specialty crop as well as characteristics of the individual farm operation. Storage capacity, farm size, financing constraints, and the management ability of the producer are just a few examples of heterogeneity with respect to specialty crop production costs across farming operations.

The survey conducted by Fulton, Pritchett, and Pederson (2003) reports that higher variable production costs, higher investment costs, and managerial time requirements were three of the top four reasons why Indiana producers chose not to grow any specialty crops. The number one reason for not producing specialty crops was the lack of a market for sale of the specialty crop. For surveyed producers who did produce specialty crops under contract, revenue enhancement and market access were the top two reasons reported for contracting.

Contract Model

The terms “specialty” and “commodity” are used to differentiate the contracted crop and the alternative cash market crop in the model. Examples would include high oil, white, or waxy corn (specialty) and #2 yellow corn (commodity). Soybean production would be another example in commodity form, with the specialty crop being DuPont’s STS[®] or tofu soybeans. Moreover, the non-GMO or organic forms of any general commodity crop would be another example of what is referred to as a specialty crop. For simplicity, bushels will be used as the production unit measure for the specialty and commodity crop. However, the model could easily be generalized to other types of crops whose production is measured in other units (i.e. tons or lbs.). In the sections that follow, subscripts denote differentiation unless otherwise noted.

Overview

The modeled contracting scenario is a familiar one in agriculture. Consider a profit-maximizing (risk-neutral) monopsonistic processor who procures specialty crop production from a group of producers by way of a production contract¹. In stage I, the processor offers a contract for the specialty crop to producers. The producers then decide to either accept the contract to produce the specialty crop, or to reject the contract and produce the commodity crop. Producers who accept the contract then move into stage II where production of the specialty crop takes place per specific management guidelines outlined in the contract². If the contract guidelines are followed the processor “verifies” the specialty crop for each contracted farmer through costly monitoring, acknowledging that his production carries the

¹ The phrase “production contract” is used as a general term encompassing the more specific acreage and bushel contract types.

² The model abstracts from the specifics of the contract regarding management requirements. These could include specific input requirements as well as guidelines for the timing of field operations.

specialty trait. Thus, there is no quality or trait uncertainty in the model. Furthermore, it is assumed that the processor will not purchase any amount of unverified specialty crop production. This assumption precludes the possibility of producers choosing to grow the specialty crop speculatively (i.e. there is no *ex ante* spot market for the specialty crop)³. Moreover, given the management guidelines outlined in the contract and the intensive monitoring done by the processor, producers do not have the ability to “shirk” and attempt to pass off the commodity crop as having the specialty trait.

In stage III the farmers harvest the specialty crop and yields are realized on each farm. Each farmer’s actual yield is private information (informational asymmetry). In stage IV *ex post* spot market transactions for the specialty crop may take place between producers, depending on the type of contract offered (acreage vs. bushel) and yield realizations (supply and demand). There will not be an *ex post* spot market under acreage contracts because producers deliver all specialty bushels produced to the processor. However, if the processor offers a bushel contract an *ex post* spot market for the specialty crop may exist as contracted farmers realizing low yields, in an effort to fulfill their bushel contracts, may purchase bushels from contracted farmers who realized high yields. The processor may also enter the *ex post* spot market for the specialty crop to purchase additional specialty crop production up to his capacity constraint. The prevailing price on the specialty crop spot market will depend

³ Note that this assumption is not critical under acreage contracts because the monopsonist would never offer a price above the salvage value for any spot market production in years when the aggregate production on contracted acres is below his operating capacity. This implies no producer would ever choose to produce the specialty crop without a contract. However, under bushel contracts there exists a positive probability that a spot market for the specialty crop will exist *ex post*, and that the price on that spot market may be higher than the premium offered by the processor in the contract because of excess demand. The assumption that the processor will only purchase specialty crop production from “verified” acres is critical to the results of the paper, as it prevents any producer from choosing to speculatively produce the specialty crop for the spot market (referred to in the industry as “wildcatting”).

on the aggregate yield over contracted acres. This will be explained further in the following sections.

In stage V the contracts are settled by the farmers delivering contracted production to the processor and receiving the compensation outlined in the contract. Finally in stage VI, processing takes place and the processor sells output to a downstream user earning a processing return on each bushel processed up to the plant capacity. The processor sells any excess (above his capacity constraint) contracted specialty crop at its salvage value on the commodity market. Figure 1 summarizes the timing of all decisions and actions in the modeled contracting scenario.

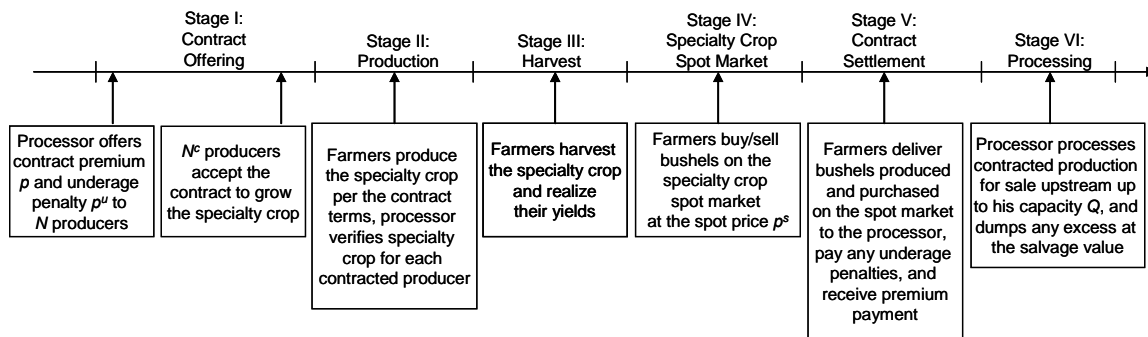


Figure 1: Timeline of the contract process

The Processor

The processor earns a net processing return R for each unit of the specialty crop processed Y , up to an exogenous capacity constraint Q^f . Thus, processing is modeled as a fixed proportions technology where each unit processed results in one unit of output (by appropriately specifying the unit of measure for the processed crop) (Carriquiry and Babcock 2004).

⁴ The problem outlined in this paper can be thought of as a short-term optimization problem for the processor. A long-term optimization problem would include the choice of the optimal plant capacity. Discussions with industry representatives in Iowa justified the assumption of an exogenous capacity constraint in a given period.

The “price” received by the processor (R) is assumed to be net of any variable production and operating costs for the processing plant. The capacity constraint Q could be thought of as a physical constraint on the technology, or due to contractual arrangements between the processor and downstream buyers (Good, Bender, and Hill 2000). Each unit of the specialty crop procured above capacity is “dumped” on the commodity market at its salvage value⁵ which is assumed to equal the price on the commodity market r less a percentage handling charge δ ⁶. The handling charge includes storage and transportation costs incurred by the processor on contracted specialty crop production that cannot be processed due to his capacity constraint.

$$(1.1) \quad R(Y;Q) = \begin{cases} RY & \text{for } Y \leq Q \\ RQ + (r - \delta)(Y - Q) & \text{for } Y > Q \end{cases}$$

where $Y = \sum_i y_i$ for all contracted farmers i

$$\delta \in [0, r]$$

To induce production of the specialty crop the processor offers a contract to producers either based on acreage or on total bushels to be delivered. The acreage contract is defined by a premium level p , above a reference price, that the processor will pay the producer for each bushel grown on contracted acres. The bushel contract is defined by a premium p above the reference price, the size of the contract in bushels y^B , and an “underage” penalty p^u that the farmer must pay to the processor if he cannot fulfill his contract in bushels of the specialty crop (because his yield fell below the contracted amount

⁵ Note that this assumption may not hold for certain specialty varieties. As an example, waxy corn cannot be sold on the #2 yellow corn market.

⁶ Net processing returns R are assumed to be net of the handling charge δ for processed bushels below the capacity constraint.

and there was not any additional production from other contracted producers available on the spot market). In addition to choosing the terms (premium and state contingent penalty) and structure (acreage or bushel) of the contract offered, the processor chooses the size of the contracting region, defined by the number of producers N within the region, where he will offer the contract⁷.

In the case of either contract type, the contract also outlines specific guidelines for the management practices that the producer must follow. These guidelines may include requirements for input use (i.e. a specific seed, fertilizers, or chemicals), storage and segregation, and timing of any production tasks. The processor enforces the management terms of the contract through costly monitoring. The costs of monitoring $m(N)$ are assumed to be an increasing convex function of the size of the contracting region, with $m_N, m_{NN} > 0$. It is assumed that as contracted farmers become spread out over a larger region it becomes increasingly difficult (costly) for the processor to monitor their management actions.

The Producers

Upon receiving the contract offer, the set of N profit maximizing (risk-neutral) producers within the contracting region choose to either accept the contract and produce the specialty crop, or produce the commodity crop for sale in the commodity market. Each producer is assumed to farm one acre and face a farm-level yield distribution for both the specialty and commodity crops, $y_i \sim f(y) \in [0, y_{max}]$ for $\forall i \in N$ (i.e. it is assumed there is no yield drag for the specialty crop) where $E[y_i] = \bar{y}$ and $Var[y_i] = \sigma_y^2$ for $\forall i$.

Furthermore, the joint distribution of farm level yields is characterized by a spatial

⁷ N could also be loosely interpreted as the number of counties in which the processor offers the contract. In reality, N may be a discrete variable, but for modeling purposes its assumed to be continuous.

correlation structure where the correlation between yields on any two farms i and j is equal to $\rho_{i,j} \geq 0$ for $\forall i \neq j$.

Commodity crop production costs c are also assumed to be homogeneous across all producers within any contracting region. Production costs for the commodity crop are equal to the sum of J variable and (annualized) fixed production costs that include labor, fertilizers, chemicals, land, machinery, fuels, storage etc. Expected profits for commodity crop production π_i^C are equal to the expected yield times the commodity price r minus production costs. The commodity price is assumed to be constant so that the analysis is focused on the effects of production uncertainty⁸.

$$(1.2) \quad E[\pi_i^C] = r\bar{y} - c \text{ for } \forall i, \text{ where } c = \sum_j x_j \text{ for } \forall i$$

Given the management guidelines outlined in the contract, production of the specialty crop will result in higher production costs (i.e. seed with special genetics, more intensive labor and management, the use of additional inputs or specific assets) than the commodity crop. Furthermore, it is assumed that these costs may vary by farm so that producers are heterogeneous with respect to specialty crop production costs under contract. The additional costs above those for the commodity crop are denoted by a non-negative additive term $\tau_{j,i}$.

$$(1.3) \quad c_i^s = \sum_j (x_j + \tau_{j,i}), \text{ where } \tau_{j,i} \geq 0 \text{ for } \forall i, j$$

Thus, specialty crop production costs are higher than for the commodity crop

$c_i^s \geq c_i = c$ for all farmers. It is also assumed that transportation and marketing costs are such

⁸ Alternatively, r could be interpreted as the expected commodity price at harvest and that specialty crop yields within the contract region are uncorrelated with the commodity market price. This interpretation would not affect the results given the risk-neutrality assumptions for both the processor and producers.

that producers, given equal prices, are indifferent between delivering bushels to the processor, selling on the specialty crop spot market, or selling on the commodity market⁹.

While each producer knows his specialty crop production cost with certainty, the processor views production costs for the specialty crop as being randomly distributed, on a non-negative and bounded support, across producers within the contracting region

$c_i^s \sim \hat{h}(c^s) \in [c, c + c^u]$, $\forall i \in N$ for $\forall N$. The distribution of specialty crop production costs across producers in any region is also common knowledge to each of the producers.

Contract Supply

Given the assumptions on the specialty grain production costs, the processor must offer premiums above the commodity price to induce farmers to accept specialty crop production contracts. By making the compensation terms of the contract more favorable the processor increases the total number of farmers in a given contracting region who will accept the contract N^c , a shift up his “contract supply curve” for a given contract region. By increasing the size of the contracting region the processor induces an outward shift to his contract supply curve increasing the number of farmers who will accept the contract for any compensation structure.

The effects of the compensation terms of the contract and the size of the region on the number of contracts accepted is given in figure 2. The left panel provides a spatial interpretation¹⁰ of the effects of increasing the size of the contract region from N_1 to N_2 ($N_2 > N_1$) on the number of accepted contracts. The right panel in figure 1 plots two conditional contract supply curves for the two contracting region sizes. The contract

⁹ These costs are the producer analogue of the processor’s handling charge δ .

¹⁰ The term spatial here is used loosely. The only spatial aspect in the model is the spatial correlation of yields across farmers. Transportation costs are assumed equal across producers.

premium is plotted on the vertical axis. Increasing the premium for a given contract size induces a move up the supply curve. Increasing the size of the contract region from N_1 to N_2 causes a rightward rotation in the processor's contract supply curve.

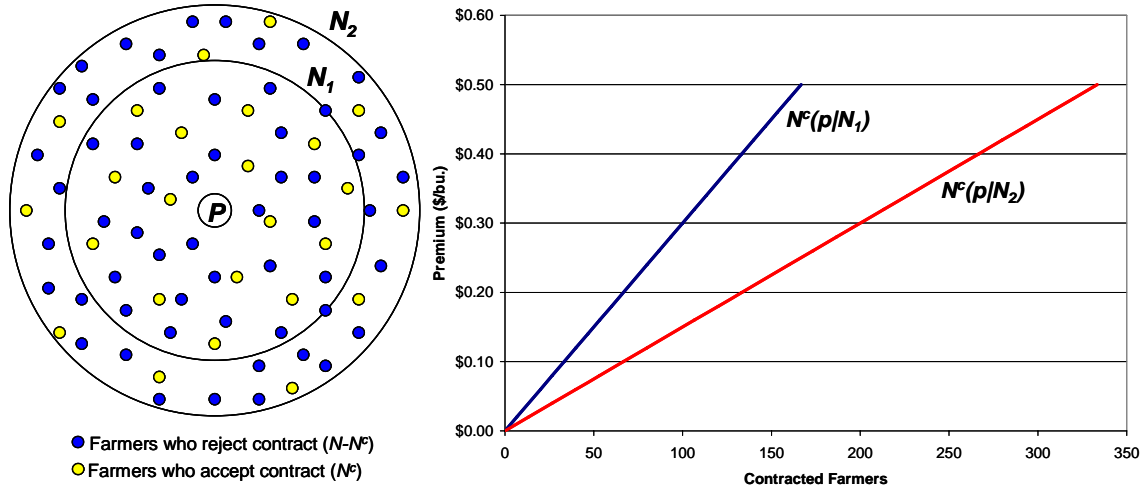


Figure 2. Effects of the premium and contract region size on the processor's contract supply curve.

In equilibrium, the processor optimally chooses the contract terms to maximize expected profits given knowledge about the profit-maximizing behavior of producers. The following subsections outline the processor and producers' problems and market equilibrium in detail under acreage and bushel contracts, respectively.

Acreage Contracts

Under acreage contracts, farmer i 's profit π_i^A is equal to his yield times the sum of the reference (commodity) price and the contract premium p less specialty crop production costs.

$$(1.4) \quad \pi_i^A = (r + p)y_i - c_i^s$$

Farmer i accepts the contract if expected profits from contracting are greater than expected profits from producing the commodity crop.

$$(1.5) \quad E[\pi_i^A] \geq E[\pi_i^C] \Rightarrow (r+p)\bar{y} - c_i^s \geq r\bar{y} - c \Rightarrow c_i^s - c = \hat{c}_i \leq p\bar{y}$$

Equation (1.5) defines the marginal producer with specialty production cost “premium” $\hat{c}^A = p\bar{y}$, where all farmers with a specialty crop production cost premium below (above) \hat{c}^A will accept (reject) the contract. Using a change of measure, the distribution of specialty crop production cost premiums within any contracting region N is given by

$$\hat{c}_i \sim h(\hat{c}_i) \in [0, c^u], \text{ where } h(\hat{c}_i) = h(c_i^s - c) = \hat{h}(c_i^s). \text{ Also, let } H(c) = \int_0^c h(x)dx = \Pr[\hat{c} \leq c]$$

denote the cumulative distribution function of the specialty crop production cost premium.

The processor offers a premium p above the commodity reference price, that will be paid for each bushel of the specialty crop grown on the N^c contracted acres, within the chosen contracting region N . Under acreage contracts there will be no *ex post* spot market for the specialty crop because each farmer delivers all bushels produced on contracted acres to the processor. The processor’s profit is equal to processing returns less procurement and diversification costs. For each bushel of the specialty crop dumped on the commodity spot market the processor incurs a handling fee δ that is expressed as a percentage of the spot price. The processor chooses the premium and the size of the contracting region to maximize expected profits, subject to each producer’s decision rule¹¹ and given his information on the farm-level yield and specialty crop production cost premium distributions.

$$(1.6) \quad \max_{p, N} E[\Pi^A(p, N)] = \int_0^Q RYdG(Y) + \int_Q^{N^c y_{max}} [RQ + (r - \delta)(Y - Q)]dG(Y) - \int_0^{N^c y_{max}} (r + p)YdG(Y) - m(N)$$

¹¹ While not explicitly motivating the model as a principal-agent problem, the processor’s consideration of producer’s actions is analogous to rationality constraints within a principal-agent framework.

subject to $p \geq 0$, $N \geq 0$, and

$$Y = \sum (y_i | E[\pi_i^A] \geq E[\pi_i^C]) \sim g(Y; p, N) \in [0, N^c y_{max}]$$

where $\hat{c}_i \sim h(\hat{c})$, $y_i \sim f(y) \forall i$

Total procurement for the processor Y is equal to the sum of the yield realizations for each contracted producer. Therefore, the distribution of total procurement g , with cumulative distribution function G , is a function of the farm level yield distribution f and the number of farmers who accept the contract, which in turn is a function of the premium offered and the size of the contracting region¹². Formally, $N^c = H(p\bar{y})N$ with $N_p^c = h(p\bar{y})N\bar{y} \geq 0$ and $N_N^c = H(p\bar{y}) \geq 0$. Thus, $G(Y; p + dp, N) \leq G(Y; p, N)$ and $G(Y; p, N + dN) \leq G(Y; p, N)$ for $\forall dp, dN \geq 0$. This implies that increasing the premium or size of the contracting region induces a shift of first-order stochastic dominance in G .

Using Leibniz rule and noting that $\int_0^{N^c y_{max}} Y dG(Y) = E(Y) = H(p\bar{y})N\bar{y}$, where $H(\cdot)$ is the cumulative distribution function for the specialty crop production cost premium, the solution to the processor's problem (p^{A*}, N^{A*}) satisfies the following first order conditions.

All functions are evaluated at the optimum unless otherwise noted.

$$(1.7) \quad \begin{aligned} \frac{\partial E[\Pi^A]}{\partial p} &= \int_0^Q RY dG_p + \int_Q^{N^c y_{max}} [RQ + (r - \delta)(Y - Q)] dG_p \\ &+ [RQ + (r - \delta)(N^c y_{max} - Q)] g(N^c y_{max}) N_p^c y_{max} \\ &- (r + p^{A*}) N_p^c \bar{y} - N^c \bar{y} \leq 0, \quad p^{A*} \geq 0 \end{aligned}$$

¹² If yields at the farm level were assumed independent, normality could be assumed as an approximation for the distribution of total procurement using the Central Limit Theorem. However, it is widely accepted that crop yields exhibit positive spatial correlation. Therefore, no assumptions are made on the functional form of g , or the distribution of yields at the farm level f . The cost of this generality, as usual, is the inability to derive explicit analytic solutions for the contract equilibria in either the acreage or bushel contract cases.

$$\begin{aligned}
(1.8) \quad \frac{\partial E[\Pi^A]}{\partial N} &= \int_0^Q RY dG_N + \int_Q^{N^c y_{max}} [RQ + (r - \delta)(Y - Q)] dG_N \\
&+ [RQ + (r - \delta)(N^c y_{max} - Q)] g(N^c y_{max}) N_N^c y_{max} \\
&- (r + p^{A*}) N_N^c \bar{y} - m_N \leq 0, \quad N^{A*} \geq 0
\end{aligned}$$

Note that both non-negativity constraints must be non-binding in any equilibrium with contracting so that the first order conditions will be strictly equal to zero. Equations (1.7) and (1.8) equate the marginal benefits to the marginal costs of increasing the premium p and the size of the contracting region N , respectively. The marginal benefits of increasing the premium are equal to the increase in processing returns, given by the first three terms in equation (1.7). Similarly, the net benefits of increasing N are given in the first three terms of equation (1.8). The third term, in both cases, reflects the fact that the upper bound of the aggregate yield distribution is increasing in the number of acres contracted by the processor (i.e. $Y_{max} = N^c y_{max}$).

The marginal costs of increasing the premium in (1.7) are equal to the increased cost on each contracted acre plus the additional cost of increasing the amount of contract acres from increasing the premium. The marginal costs of increasing the size of the contract region in (1.8) are equal to the increase in procurement costs as more contracts will be accepted for any given premium, plus the increase in monitoring costs from expanding the size of the contracting region. The second order sufficient conditions for a maximum are given in the Appendix and are assumed to hold.

Bushel Contracts

Given a bushel contract offer, farmer i 's profit π_i^B is equal to the reference price plus the premium times the size of the contract y^B , less production costs for the specialty crop.

For simplicity, the analysis is limited to bushel contracts where $y^B = \bar{y}$. This of course implies that aggregate contracted bushels will equal $N^c \bar{y} = \bar{Y}$. When actual yield is less than the size of the contract the farmer must either pay a fixed underage penalty (specified in the contract offer) on each unit below the contracted amount, or enter the specialty crop spot market (if there is positive supply) to purchase excess production from other contracted farmers to fulfill his contract obligation. When farmer i 's yield is greater than the contract size he can sell the excess specialty crop production at its salvage value on the commodity market or sell it on the specialty crop spot market (if demand exists).

$$(1.9) \quad \pi_i^B = (r + p)\bar{y} + I(y_i) p^s(Y, p^u, r | y_i > \bar{y})(y_i - \bar{y}) \\ + [1 - I(y_i)] p^s(Y, p^u, r | y_i \leq \bar{y})(y_i - \bar{y}) - c_i$$

$$\text{where } I(y_i) = \begin{cases} 1 & \text{if } y_i > \bar{y} \\ 0 & \text{otherwise} \end{cases}$$

The prevailing price on the specialty crop spot market p^s will depend on the underage penalty p^u set by the processor in the bushel contract, the salvage value r , and the aggregate specialty crop production Y across all contracted farmers. At the time of contract signing, producers use their knowledge of the distribution of production cost premiums for the specialty crop and their information on the joint distribution of yields within the contracting region to formulate an expectation for the *ex post* specialty crop spot market price. There are two possible scenarios. The aggregate yield realization will either be equal to or below the total contracted by the processor $Y \leq \bar{Y}$, or greater than the contracted amount $Y > \bar{Y}$. If $Y \leq \bar{Y}$, excess demand on the specialty crop spot market will bid the spot price up to the

underage penalty¹³, $p^s = p^u$. If $Y > \bar{Y}$ there will be excess supply on the specialty crop spot market and the prevailing spot price will equal the salvage value of the commodity market price, $p^s = r$.

In either aggregate yield case, farmer i 's yield could be greater than or less than the individual contract size, $y_i > \bar{y}$ or $y_i \leq \bar{y}$, creating four possible *ex post* spot market scenarios under bushel contracts. Farmer i 's expectations for the prevailing specialty crop spot price conditional on his yield falling above or below the contract size are given below.

$$(1.10) \quad E[p^s | y_i \leq \bar{y}] = p^{s,u} = \theta_1 p^u + (1 - \theta_1)r \leq p^u$$

$$(1.11) \quad E[p^s | y_i > \bar{y}] = p^{s,o} = \theta_2 p^u + (1 - \theta_2)r \geq r$$

$$\text{where} \quad \begin{aligned} \theta_1 &= \Pr[Y \leq \bar{Y} | y_i \leq \bar{y}] \\ \theta_2 &= \Pr[Y \leq \bar{Y} | y_i > \bar{y}] \end{aligned}$$

The conditional probabilities θ_1 and θ_2 will depend on how farmer yields are jointly distributed across the contracting region (i.e. the spatial correlation), with $\theta_1 \geq (\leq) \theta_2$ when yields are positively (negatively) correlated. If yields are perfectly correlated there will not be an *ex post* spot market because farmers will either pay the underage penalty p^u to the processor when (all) yields are below the mean, or sell the excess specialty crop on the commodity market at the salvage value r when (all) yields are above the mean (i.e. $\theta_1 = 1$, $\theta_2 = 0$). If yields are independent, $\theta_1 = \theta_2 = \frac{1}{2}$ and the expected spot price is $\frac{1}{2}(p^u + r)$ in

¹³ Note that if the aggregate yield realization is such that only one farmer would not be able to fulfill his contract obligation with purchases on the spot market the prevailing spot market price would not be p^u . There is no solution in this specific case where there are a limited number of buyers bidding for a fixed supply. This is the same problem faced by Carriquiry and Babcock (2004). For simplicity, it is assumed that farmers expect the price on the specialty crop spot market to be bid up to p^u for all excess demand scenarios.

both the underage and overage scenarios. This implies that if yields are independent, the magnitude of the underage penalty has no effect on farmer participation because the expected cost of the penalty when farmer i 's yield is below the mean is exactly equal to the benefits of the penalty when his actual yield is above average. This is purely a result of farmer expectations for the *ex post* specialty crop spot price.

Given farmer i 's expectation for the *ex post* specialty crop spot market price, expected profit for the bushel contract is given below in equation (1.12).

$$\begin{aligned}
 E[\pi_i^B] &= (r + p)\bar{y} - c_i^s - p^{s,u} \int_0^{\bar{y}} (\bar{y} - y) dF(y) + p^{s,o} \int_{\bar{y}}^{y_{max}} (y - \bar{y}) dF(y) \\
 (1.12) \quad &= (r + p)\bar{y} - c_i^s - \{\theta_1 p^u + (1 - \theta_1)r\} \Delta y + \{\theta_2 p^u + (1 - \theta_2)r\} \Delta y \quad , \\
 &= (r + p)\bar{y} - c_i^s + (r - p^u)(\theta_1 - \theta_2) \Delta y
 \end{aligned}$$

where $\Delta y = \int_0^{\bar{y}} (\bar{y} - y) dF(y) = \int_{\bar{y}}^{y_{max}} (y - \bar{y}) dF(y)$ by the definition of \bar{y} . Thus, if yields are positively correlated, the farmer will pay less (in expectation) than p^u in the case of an individual underage and will receive a price greater (in expectation) than the salvage value r in the case of an individual overage¹⁴.

Equation (1.12) defines the marginal production cost premium for the specialty crop \hat{c}^B , where all producers with $\hat{c}_i \leq (>) \hat{c}^B$ will accept (reject) the bushel contract offered by the processor. Comparing this to the marginal producer under acreage contracts, defined by \hat{c}^A , any underage penalty that is greater than the commodity price will require a greater bushel contract premium to induce the same level of farmer contract acceptance relative to the acreage contract (for a given N).

¹⁴ This is assuming $p^u \geq r$ in equilibrium, which is shown to hold, and that yields are positively correlated across space.

$$(1.13) \quad \hat{c}^B = p\bar{y} + (r - p^u)(\theta_1 - \theta_2)\Delta y$$

Given the profit-maximizing decision rule of the producers, the processor chooses the premium level p and the underage penalty p^u to maximize expected profits. In equilibrium there must be some constraints on the values of the underage penalty. If the processor sets the underage penalty to a value below the commodity price, each contracted producer has an incentive to report zero yield (private information) to the processor and pay the underage fee while selling the specialty crop on the commodity market. This nets each producer the reference price plus premium times the contract size (a sort of lump sum payment), plus the difference between the underage penalty and reference price for each bushel produced up to the contract size (or actual yield if it is below the mean), plus the reference price for each unit produced above the mean. The maximum underage penalty that the processor can charge is assumed to be equal to his net processing return R^{15} . For any underage penalty greater than the net processing margin, the processor would be better off collecting the underage than accepting delivery of contracted bushels for processing. As an arbitrage condition, it is assumed producers would simply not accept such a contract.

The processor's maximization problem for bushel contracts is presented formally below in (1.14). The first order conditions are obtained by differentiating the processor's constrained objective function using Leibniz rule, and are presented in (1.15)-(1.19) where λ and μ are the Lagrange (L) multipliers for the inequality constraints on the underage penalty. The second order conditions for the processor's problem with bushel contracts, which are assumed to hold, are provided in the Appendix.

¹⁵ Again, from a principal-agent perspective, the constraints on the underage penalty would be analogous to incentive compatibility constraints.

$$(1.14) \max_{p, N, p^u} E[\Pi^B(p, N, p^u)] = \int_0^Q (R-r)YdG(Y) + \int_Q^{N^c y_{max}} (R-r)QdG(Y) - pN^c \bar{y} \\ + \int_0^{N^c \bar{y}} p^u (N^c \bar{y} - Y)dG(Y) - m(N)$$

subject to $p, N \geq 0$, $r \leq p^u \leq R$, and

$$Y = \sum(y_i | E[\pi_i^B] \geq E[\pi_i^C]) \sim g(Y; p, N)$$

where $\hat{c}_i \sim h(\hat{c})$, $y_i \sim f(y) \forall i$

$$(1.15) \frac{\partial L}{\partial p} = \int_0^Q (R-r)YdG_p + \int_Q^{N^c y_{max}} (R-r)QdG_p + (R-r)Qg(N^c y_{max})N_p^c y_{max} \\ - p^{B*} N_p^c \bar{y} - N^c \bar{y} + p^{u*} \left\{ \int_0^{N^c \bar{y}} (N^c \bar{y} - Y)dG_p + \int_0^{N^c \bar{y}} N_p^c \bar{y} dG \right\} \leq 0, p^{B*} \geq 0$$

$$(1.16) \frac{\partial L}{\partial N} = \int_0^Q (R-r)YdG_N + \int_Q^{N^c y_{max}} (R-r)QdG_N + (R-r)Qg(N^c y_{max})N_N^c y_{max} \\ - p^{B*} N_N^c \bar{y} - m_N + p^{u*} \left\{ \int_0^{N^c \bar{y}} (N^c \bar{y} - Y)dG_N + \int_0^{N^c \bar{y}} N_N^c \bar{y} dG \right\} \leq 0, N^{B*} \geq 0$$

$$(1.17) \frac{\partial L}{\partial p^u} = \int_0^Q (R-r)YdG_{p^u} + \int_Q^{N^c y_{max}} (R-r)QdG_{p^u} \\ + (R-r)Qg(N^c y_{max})N_{p^u}^c y_{max} - p^{B*} N_{p^u}^c \bar{y} + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y)dG \\ + p^{u*} \left\{ \int_0^{N^c \bar{y}} (N^c \bar{y} - Y)dG_{p^u} + \int_0^{N^c \bar{y}} N_{p^u}^c \bar{y} dG \right\} + \lambda^* - \mu^* = 0$$

$$(1.18) \frac{\partial L}{\partial \lambda} = R - p^{u*} \geq 0, \lambda^* \geq 0, \text{ and } \lambda^* [R - p^{u*}] = 0$$

$$(1.19) \frac{\partial L}{\partial \mu} = p^{u*} - r \geq 0, \mu^* \geq 0, \text{ and } \mu^* [p^{u*} - r] = 0$$

As in the acreage contract solution, both of the non-negativity constraints on p and N will not bind for any interior solution with contracting. Conditions (1.15)-(1.17) equate the marginal benefits of increasing the premium, size of the contracting region, and the underage penalty to their marginal costs. The marginal benefits of increasing the premium and size of

the contracting region are equal to the expected marginal increases in revenues from processing returns, while the marginal costs are equal to the expected increases in procurement and monitoring costs. The marginal benefits of increasing the underage penalty are equal to the expected increase in underage penalties and the reduction in procurement costs due to lower farmer acceptance. The marginal costs of increasing the underage penalty are equal to the expected reduction in processing returns due to lower farmer acceptance of the contract. Conditions (1.18) and (1.19) ensure that the constraints are satisfied at the optimal solution.

Analytic Results

While the generality of the model precludes the derivation of explicit analytical solutions for the optimal acreage and bushel contract equilibriums, some claims can still be made regarding the two contract structures. The first result shows that bushel contracts can Pareto dominate acreage contracts.

Proposition 1: There exists a bushel contract which Pareto dominates the optimal acreage contract, although that bushel contract may not be the processor's optimal bushel contract.

Proof:

Consider a bushel contract where the premium and size of the contract region are set equal to the values of the optimal solution for the acreage contract, and the underage penalty is set equal to the salvage value r . Note that farmer acceptance of the two contracts will be equal under these conditions, so that expected farmer profits are the same for the optimal acreage

and proposed bushel contracts (i.e. farmers are no worse off)¹⁶. Given equal contract acceptance, the aggregate distribution of contracted production will be the same so that the processor's expected profits can be compared under the two contract specifications.

Subtracting the processor's expected profit under the optimal acreage contract from expected profits under the proposed bushel contract yields the desired result that the processor is at least as well off under the proposed bushel contract than the optimal acreage contract.

Formally,

$$E\left[\Pi^B(p^{A^*}, N^{A^*}, p^u = r)\right] - E\left[\Pi^A(p^{A^*}, N^{A^*})\right] = \delta \int_Q^{N^c y_{max}} [Y - Q] dG \geq 0.$$

Moreover, if the handling charge δ is strictly greater than zero, the processor's expected profits under the bushel contract are strictly greater than expected profits under the optimal acreage contract. Therefore, the proposed bushel contract Pareto dominates the optimal acreage contract. Finally, $E\left[\Pi^B(p^{B^*}, N^{B^*}, p^{u^*})\right] \geq E\left[\Pi^B(p^{A^*}, N^{A^*}, p^u = r)\right]$ by the definition of a maximum. This trivially proves that the proposed bushel contract may not be optimal.

The intuition behind Proposition 1 is that by offering an equivalent (as far as farmers are concerned) bushel contract, the processor is able to avoid handling excess specialty crop production in years with above average yield realizations. This result relies heavily on the assumptions that 1) farmers are indifferent between delivering bushels to the processor and selling them on the commodity market, and 2) the salvage value is equal to the commodity

¹⁶ Note that this result would also hold for risk-averse producers, at least in a mean-variance framework. By definition, expected producer profits are equal under the proposed bushel and optimal acreage contracts, while the variance of expected profits under the proposed bushel contract are actually smaller than the optimal acreage contract $\left(\left((r + p^{A^*})\sigma_y\right)^2 \geq (r\sigma_y)^2 \text{ for } p^{A^*} \geq 0\right)$.

price r . Note that under the proposed bushel contract, the producer is guaranteed the premium p on each bushel contracted. For aggregate yield realizations below the contracted level the processor realizes higher procurement costs than those in the optimal acreage contract because he is paying the premium for the total contract size, but only being reimbursed r for each short bushel in the case of an aggregate yield shortage. This represents the “price” the processor must pay for the farmer to take on the risk of yield realizations below the bushel contract size.

A corollary to Proposition 1 is that producers will prefer the bushel contract proposed in Proposition 1 to the optimal acreage contract with $\delta \geq 0$. This can be formally stated as:

$$(1.20) \quad \pi_i^B(p^{A^*}, N^{A^*}, p^u = r) \geq \pi_i^A(p^{A^*}, N^{A^*}; \delta \geq 0) \text{ for } \forall i$$

The proof of this claim is straightforward. Total differentiation of the first order conditions for the acreage contract with respect to δ yields the following comparative static results.

$$(1.21) \quad \frac{\partial p^{A^*}}{\partial \delta} = \frac{\int_Q^{N^c y_{max}} (Y - Q) dG_p}{\frac{\partial^2 \Pi^A}{\partial p^2}} \leq 0$$

$$(1.22) \quad \frac{\partial N^{A^*}}{\partial \delta} = \frac{\int_Q^{N^c y_{max}} (Y - Q) dG_N}{\frac{\partial^2 \Pi^A}{\partial N^2}} \leq 0$$

The denominators of (1.21) and (1.22) are negative by the second order conditions, while the numerators are both non-negative given increases in p and N induce shifts of first-order stochastic dominance in the distribution function G . Thus, both the premium and size of the contracting region are larger under acreage contracts when $\delta = 0$ implying farmer participation and expected producer profits are also greater when $\delta = 0$.

Noting that the proposed bushel contract is equivalent to the optimal acreage contract when $\delta = 0$, expected producer profits are greater under the proposed bushel contract than under the optimal acreage contract when $\delta > 0$, or

$$\pi_i^B(p^{A^*}, N^{A^*}, p^u = 1) = \pi_i^A(p^{A^*}, N^{A^*}; \delta = 0) \geq \pi_i^A(p^{A^*}, N^{A^*}; \delta \geq 0).$$

Finally, a claim can be made on the optimal bushel contract underage penalty. Except for very specific conditions, one of the constraints on the optimal underage penalty will bind.

$$\text{Proposition 2: The optimal underage penalty equals } r(R) \text{ if } \left[\frac{\int_0^{\bar{Y}} (\bar{Y} - Y) dG}{(\theta_1 - \theta_2) N^c \Delta y} - 1 \right] < (>) 0.$$

Proof:

Suppose at the optimum that the constraints on the optimal underage penalty do not bind,

$p^{u^*} \in (r, R)$. Define a simultaneous change in the bushel contract premium and the underage penalty such that farmer acceptance is held constant (i.e. the marginal specialty crop production cost premium is held constant).

$$(1.23) \quad d\hat{c}^{B^*} = [\bar{y}] dp - [(\theta_1 - \theta_2) \Delta y] dp^u = 0 \Rightarrow dp = \frac{(\theta_1 - \theta_2) \Delta y}{\bar{y}} dp^u$$

Then the change in the processor's expected profits for the simultaneous change in the

premium and underage penalty is given by $\left[\int_0^{\bar{Y}} (\bar{Y} - Y) dG - (\theta_1 - \theta_2) N^c \Delta y \right] dp^u$. Thus, if

$$\left[\frac{\int_0^{\bar{Y}} (\bar{Y} - Y) dG}{(\theta_1 - \theta_2) N^c \Delta y} - 1 \right] > 0 \text{ and the underage penalty is less than } R, \text{ the processor can increase}$$

expected profits by increasing the underage penalty to R while simultaneously increasing the premium according to (1.23), which violates an optimum for $p^{u^*} \in (r, R)$. The same logic

holds when $\left[\frac{\int_0^{\bar{Y}} (\bar{Y} - Y) dG}{(\theta_1 - \theta_2) N^c \Delta y} - 1 \right] < 0$, in that expected profits will increase if the underage

penalty is reduced to r while the premium is simultaneously reduced according to (1.23), again a violation of the supposition of a maximum with $p^{u*} \in (r, R)$.

The intuition behind Proposition 2 lies in what the ratio $\frac{\int_0^{\bar{Y}} (\bar{Y} - Y) dG}{(\theta_1 - \theta_2) N^c \Delta y}$ represents.

The numerator reflects the marginal valuation of the underage penalty to the processor. Similarly, the denominator reflects the marginal cost of the underage penalty for the producers. If the marginal value to the processor is greater than the aggregate marginal cost to producers, the optimum is attained at the maximum underage penalty. An analogous argument holds when the marginal valuation of the underage penalty to the processor is less than the aggregate marginal cost to producers.

Numerical Example

Since an explicit analytical solution does not exist for the model, a numerical approach was used to solve the model given functional form assumptions for the farm level yield and specialty crop production cost premium distributions, and the monitoring cost function. Specialty crop production cost premiums were assumed to follow a uniform distribution, $\hat{c}_i \sim U[0, c^u]$. A simple quadratic form was assumed for the monitoring cost function.

$$(1.24) \quad m(N) = \frac{\beta}{2} N^2$$

The three parameter beta distribution, described by (1.25), was chosen for the yield distribution because it can easily be parameterized to have finite bounds and to be either symmetric around the mean, left skewed, or right skewed. Moreover, the three parameter beta distribution has previously been used to approximate crop yield distributions (Babcock and Hennessy 1996). The beta distribution was calibrated so that farm level yields had a mean of 100 and a coefficient of variation of 20%. This level of yield volatility is consistent with federal crop insurance rates for corn and soybeans in many counties throughout the Midwest.

$$(1.25) \quad f(y) = \frac{\Gamma[a+b]}{\Gamma[a]\Gamma[b]} \frac{(y)^{a-1}(y_{max}-y)^{b-1}}{y_{max}^{a+b-1}}, \quad 0 \leq y \leq y_{max}$$

A baseline parameterization for an acreage contract with an optimal premium and contracting region size of 0.2 and 750, respectively, was achieved by solving the model with yields fixed at their means. The commodity price was set equal to one, the average yield level was set to 100, and the upper bound on the specialty crop production cost premium was set equal to 150. With fixed yields, the first order conditions (1.7) and (1.8) for the optimal acreage contract reduce to equations (1.26) and (1.27), which solve for the net processing return R and the diversification cost function parameter β in terms of the other model parameters and the desired optimal contract premium and region size. The plant capacity Q was then set equal to the optimal aggregate procurement level with yields fixed at their means. A summary of the calibrated parameters for the baseline case is given in table 1.

$$(1.26) \quad R = 1 + 2p$$

$$(1.27) \quad \beta = N \frac{c^u}{(p\bar{y})^2}$$

Table 1. Baseline Parameter Values

Parameter	Value	Parameter	Value
\bar{y}	100	R	1.4
c^u	150	r	1
β	0.00355	a	12
Q	10000	b	12
δ	0.00	y_{max}	200

The solution to the processor's profit maximization problem was then solved under yield uncertainty using numerical methods. Four different yield correlation structures were examined, ranging from independent to perfectly correlated yields¹⁷. A resorting method based on rank correlations was used to impose the desired level of correlation between individual farmer yield draws (Iman and Conover 1982). Additionally, model results were also calculated assuming the volatility of farm level yields was 40% to examine the effects of increasing yield volatility at the individual farm level.

Table 2 reports the optimal acreage contract terms for the baseline case. Compared to when yields are fixed at their mean (reported in the second column), the processor offers an acreage contract with a lower premium to fewer producers when yield uncertainty is introduced. The optimal premium decreases from \$0.197 to \$0.182 as yields become more correlated, while the number of contracted producers falls by more than 20% from nearly 96 to 76.

¹⁷ A (possibly) more realistic assumption would be that the correlation structure was a function of the size of contracting region N and the distance between farming operations. However, this would have required estimation (and possible misspecification) of a relationship between distance and correlation of yields as well as significantly increased the computing time needed for solution convergence. The simpler approach was adopted because of a lack of farm-level data for specialty crop yields.

The introduction of yield uncertainty exposes the processor to the “risk” of above average yield realizations and having to handle grain in excess of his processing capacity. This effect increases as the individual yields become more positively correlated. This is due to the direct relationship between the volatility of total procurement and the spatial correlation of yields. When yields are independent, low yield realizations are balanced by above average yields on other farms. In short, the processor is able to pool the production risk. At the other extreme, when yields are perfectly correlated, the aggregate procurement of the processor is extremely volatile and the probability of the processor being obligated to purchase production in excess of plant capacity increases. In this extreme case, production risk is purely systemic and the processor is unable to pool any of the production risk across contracted farmers.

With acreage contracts the processor earns a negative profit margin, equal to the acreage contract premium plus the handling charge, on every bushel procured above capacity. To insure against these losses, the processor reduces the premium and size of the contracting region to reduce the chance of having to operate above capacity. This results in fewer farmers accepting the contract, implying a reduction in farmer profits when yield uncertainty is introduced. This effect is magnified as yield risk becomes increasingly systemic. The last row of table 2 illustrates this effect, showing that the additional profits earned by producers declines as the level of correlation between farm level yields increases.

Table 3 reports the optimal acreage contract parameters when the processor’s handling fee δ for procurement above his plant capacity is equal to 0.10. The 10% handling charge implies even larger losses on every bushel procured above capacity compared to the situation in table 2. The resulting optimal acreage contract is characterized by a slightly

lower premium, and is offered to fewer total farmers resulting in a lower level of farmer participation and a reduction in expected profits for both the processor and the producer for all levels of yield correlation. Again, the last row of table 3 illustrates the decline in additional profits earned by the producers through contracting as the poolability of yield risk declines.

Table 2. Optimal Acreage Contracts, $\sigma_y = 20$, $\delta = 0$

$\rho_{i,j}$	$y_i = \bar{y}$	$y_i \sim f(y) = \text{Beta}(a, b, y_{max})$			
	-	0.00	0.50	0.80	1
p^{A*}	0.2	0.1971	0.1866	0.1844	0.1828
N^{A*}	750	728.43	652.91	637.91	626.62
N^c	100	95.72	81.22	78.42	76.36
$E[\Pi]$	1000	998.38	966.59	953.80	945.74
$E[\sum_i(\pi_i^A - \pi_i^c)]$	1000	943.32	757.78	723.03	697.93

Table 3. Optimal Acreage Contracts, $\sigma_y = 20$, $\delta = 0.10$

$\rho_{i,j}$	$y_i = \bar{y}$	$y_i \sim f(y) = \text{Beta}(a, b, y_{max})$			
	-	0.00	0.50	0.80	1
p^{A*}	0.2	0.1970	0.1861	0.1837	0.1820
N^{A*}	750	727.74	649.15	632.91	621.02
N^c	100	95.58	80.54	77.51	75.35
$E[\Pi]$	1000	998.30	964.60	950.73	942.45
$E[\sum_i(\pi_i^A - \pi_i^c)]$	1000	941.46	749.42	711.93	685.69

The optimal bushel contract parameters, when farm level yields have a 20% coefficient of variation, are reported in table 4 over a range of yield correlation levels. Using the bushel contract structure, the processor is able to eliminate the risk of procuring a production level above his plant capacity. Moreover, the processor can set an underage

penalty to recover profits when aggregate production is less than the total contracted. When aggregate production is greater than the total contracted, the processor can realize even greater profits by purchasing any excess (up to his capacity constraint) at the salvage value (commodity price) from contracted producers on the *ex post* spot market. This effectively shifts the majority of production risk on to the producers. However, producers are compensated for taking on a greater portion of the yield risk through higher contract premiums relative to the acreage contract structure for any given yield correlation structure. For example, when the correlation between farm yields is 0.8 and there is no handling charge, the acreage contract equilibrium results in 78.42 contracted farmers with a premium of 0.1844. The bushel contract equilibrium under the same conditions results in 85.22 farmers under contract at a premium of 0.2183.

Table 4. Optimal Bushel Contracts, $\sigma_y = 20$

$\rho_{i,j}$	$y_i = \bar{y}$	$y_i \sim f(y) = \text{Beta}(a, b, y_{max})$			
	-	0.00	0.50	0.8	1
p^{B*}	0.20	0.1956	0.2183	0.2183	0.2139
N^{B*}	750	753.53	654.12	654.12	694.24
N^c	100	97.16	88.03	85.22	84.87
$E[\Pi]$	1000	1027.70	1208.40	1240.23	1202.46
p^{u*}	-	1.400	1.400	1.400	1.299
$p^{s,u}$	-	1.200	1.303	1.342	1.299
$p^{s,o}$	-	1.200	1.099	1.058	1.000
$E\left[\sum_i (\pi_i^B - \pi_i^c)\right]$	1000	944.27	888.55	832.74	833.82

When yields are independent so that production risk is poolable, the increase in the processor's expected profits from using bushel contracts is minimal. Expected profits under

bushel contracts and independent yields was estimated to be \$1027.70, an increase of only \$30 compared to both acreage contract scenarios reported. However as the level of correlation between yields is increased, and production risk becomes increasingly systemic, the processor is able to extract even greater relative gains from using bushel contracts. In fact, the processor is able to earn greater expected profits using bushel contracts when yields are uncertain compared to when yields are fixed at the mean.

For example, expected processor profits for bushel contracts were estimated to be over \$1,200 when yields are uncertain and positively correlated compared to only \$1,000 in the case of certain yields. This is because of the *ex post* spot market that is created by bushel contracts. When aggregate production exceeds the total contracted, the processor is able to enter the spot market and purchase the excess at the commodity price, earning an even greater profit margin on each bushel in excess of the total contracted (up to his capacity constraint). These results imply that bushel contracts may be more prevalent when processors are unable to pool production risk across contracted producers. This may be the case for certain crops or geographic regions. Comparing producer profits to the corresponding values in table 2 shows that the producers also earn greater profits, in expectation, under bushel contracts.

Table 4 also reports the prices on the specialty crop spot market expected by producers when farm-level yield is below or above the contracted amount (mean yield). In the case of an individual underage (overage), the producer's expectation for the spot price $p^{s,u}$ ($p^{s,o}$) ranges from 1.20 (1.20) when yields are independent to 1.34 (1.058) when the correlation between farm level yields is equal to 0.80. The last row of table 4 reports additional profits earned by the producers through contracting.

When yields are perfectly correlated, the expected spot price in the case of a farm-level underage equals the underage penalty of 1.4, and is equal to the commodity price of one in the case of an above average yield realization at the farm level. Farmers expectations of the *ex post* spot price depend only on their own yield distribution, which is the aggregate distribution when yields are perfectly correlated. The farmer's expectations do not dampen the underage penalty. The processor then chooses a lower underage penalty compared to the cases of positively, but not perfectly correlated, yields and expected profits fall.

Tables 5-7 report the numerical results when farm yield volatility is doubled to 40%. Increasing volatility at the farm level increases the volatility of aggregate contracted production for all levels of yield correlation. The processor offers acreage contracts with a lower premium to an even smaller group of producers when the farm level yield volatility is increased. Similar to the baseline volatility case, when a positive handling fee is imposed the processor further reduces the premium and contracting region size to contract with a smaller group of producers to reduce the magnitude and probability of losses when yields are above average, although the effects are relatively small. Additionally, increased farm-level yield volatility increases the expected costs of an underage to the producer, requiring either a higher premium or lower underage penalty to keep participation constant.

However, increased farm level yield volatility actually increases expected profits, relative to the baseline case, when the processor uses bushel contracts. Table 7 shows that expected profits increase over \$100 relative to the 20% yield volatility solution. Again, this is illustrating the benefit of the *ex post* specialty crop spot market to the processor. When yields are more volatile at the farm level, aggregate contracted production will also be more volatile. The processor optimally contracts with fewer farmers by reducing the premium and

size of the contracting region, and is able to take advantage of above average aggregate production by purchasing the specialty crop on the spot market at the commodity price up to his capacity constraint.

Table 5. Optimal Acreage Contracts, $\sigma_y = 40$, $\delta = 0$

$\rho_{i,j}$	$y_i = \bar{y}$	$y_i \sim f(y) = \text{Beta}(a, b, y_{max})$			
	-	0.00	0.50	0.80	1
p^{A*}	0.2	0.1947	0.1779	0.1742	0.1719
N^{A*}	750	710.77	593.73	568.98	554.07
N^c	100	92.26	70.42	66.08	63.50
$E[\Pi]$	1000	994.63	916.95	891.83	876.44
$E[\sum_i(\pi_i^A - \pi_i^c)]$	1000	898.15	626.39	575.56	545.78

Table 6. Optimal Acreage Contracts, $\sigma_y = 40$, $\delta = 0.10$

$\rho_{i,j}$	$y_i = \bar{y}$	$y_i \sim f(y) = \text{Beta}(a, b, y_{max})$			
	-	0.00	0.50	0.80	1
p^{A*}	0.2	0.1946	0.1768	0.1732	0.1707
N^{A*}	750	709.84	586.28	562.72	546.29
N^c	100	92.09	69.10	64.98	62.17
$E[\Pi]$	1000	994.32	912.61	886.37	870.83
$E[\sum_i(\pi_i^A - \pi_i^c)]$	1000	896.04	610.84	562.73	530.62

When yields are perfectly correlated, farmers expect *ex post* spot prices to be equal to the underage penalty or the salvage value depending on aggregate yields, which are equal to farm level yields. There is no dampening effect on the spot price expectations of farmers so the processor must lower the underage penalty and realizes smaller expected profits compared to the cases of positively, but not perfectly, correlated yields. Again, comparing

the last row in tables 5-7 shows that producers also prefer the bushel contract structure because they earn greater expected profits compared to either acreage contract scenario.

Table 7. Optimal Bushel Contracts, $\sigma_y = 40$

$\rho_{i,j}$	$y_i = \bar{y}$	$y_i \sim f(y) = Beta(a, b, y_{max})$			
	-	0.00	0.50	0.8	1
p^{B*}	0.20	0.1987	0.1987	0.1986	0.1984
N^{B*}	750	735.50	735.50	735.44	644.25
N^c	100	95.39	80.81	74.25	67.55
$E[\Pi]$	1000	1065.83	1352.32	1363.11	1310.83
p^{u*}	-	1.400	1.400	1.400	1.169
$p^{s,u}$	-	1.20	1.303	1.343	1.169
$p^{s,o}$	-	1.20	1.099	1.058	1.000
$E[\sum_i (\pi_i^B - \pi_i^c)]$	1000	927.51	666.15	612.07	623.16

Conclusions

The fact that vertical coordination through contractual relationships in agriculture is becoming increasingly important is well documented. The rise of vertical coordination in agriculture has been more apparent in livestock markets, which is reflected in the academic literature. Many authors have explored the effects of varying contract structures using theoretical, empirical, and experimental approaches. Special consideration has been given to the effect of compensation structures on the efficiency of contract market equilibriums and the co-existence of contract and spot markets for the same commodity. However, previous studies have focused more heavily on contracting in livestock markets, while production contracts in crop production have been given much less attention.

This paper makes a contribution in this area by presenting a comparison of contract structures within a theoretical model of a contracting relationship between a risk-neutral monopsonistic processor and risk-neutral producers. The main analytical result is that there exists a bushel contract structure which Pareto dominates the optimal acreage contract. However, this bushel contract may not be optimal. This result departs from the conventional thinking in the contract literature that acreage contracts will be the preferred choice of contract structure.

Furthermore, it is shown that the magnitude of the optimal underage penalty for bushel contracts depends on the relative marginal valuations of low yield realizations for the processor and producers, which are themselves functions of the spatial correlation of yields, or the poolability of production risk. Moreover, the (expected) magnitude of the underage penalty is dampened at the farmer level because of the producers' expectations of *ex post* spot market prices for the specialty crop under bushel contracts, which are conditional on the aggregate production of the specialty crop.

A calibrated numerical example shows that acreage contract premiums, farmer participation, and expected profits for both the processor and producers decline as the correlation between farm level yields across space increases. Increasing the level of yield volatility at the farm level results in the same type of effect. Numerical solutions for the optimal bushel contract under different assumptions for the spatial correlation of yields and farm level yield volatility illustrate the analytic result that the processor will prefer bushel contracts (greater expected profits) and that they may Pareto dominate acreage contracts.

The bushel contract structure allows the processor to contract with a greater number of producers at higher premium rate, increasing farmer profits relative to the acreage contract

equilibrium. As production risk becomes more systemic (larger correlation between yields), the processor benefits relatively more from using bushel contracts. When risk is largely poolable, the two contract structures are nearly equivalent with respect to expected profits and farmer participation in the resulting equilibriums. These results imply that bushel contracts may be more prevalent for crops and regions where the nature of production risk is highly systemic. When production risk can be pooled, the choice of contract structure may be less important.

These results must be interpreted with care. The model includes a set of restrictive assumptions including producer risk-neutrality, an exogenous commodity market with no price uncertainty, and the ability to dump the specialty crop on the commodity market at the commodity price. Thus, the model can be thought of as a starting point, providing for a multitude of possible extensions for future research. Obviously, producer risk aversion and commodity market price uncertainty are two potential areas for further analysis. Risk averse producers will, in general, require more compensation for taking on a larger share of production risk, potentially eroding away the gains from bushel contracts. The addition of price uncertainty may also greatly affect the results, especially if the commodity price is assumed to be correlated with aggregate or farm-level yields. Furthermore, extending the model to an oligopsony setting where multiple processors compete in production contracts would reduce the ability of the processor to increase profits using the *ex post* spot market when bushel contracts are offered.

While the limitations of the current model are well recognized, the results of this analysis do provide circumstances where bushel contracts are strictly preferred by all agents.

Empirical testing of the validity of these assumptions in real-world contract markets is another area for further research.

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Appendix

The second order sufficient conditions for the processor's problem with acreage contracts.

$$H^A = \begin{bmatrix} \frac{\partial^2 \Pi^A}{\partial p^2} & \frac{\partial^2 \Pi^A}{\partial p \partial N} \\ \frac{\partial^2 \Pi^A}{\partial N \partial p} & \frac{\partial^2 \Pi^A}{\partial N^2} \end{bmatrix} \leq 0, \text{ where}$$

$$\begin{aligned} \frac{\partial^2 \Pi^A}{\partial p^2} &= \int_0^Q RY dG_{pp} + \int_Q^{N^c y_{max}} [RQ + (r - \delta)(Y - Q)] dG_{pp} \\ &+ [RQ + (r - \delta)(N^c y_{max} - Q)] \left\{ \begin{array}{l} g_p(N^c y_{max}) N_p^c y_{max} \\ g_p(N^c y_{max}) (N_p^c y_{max})^2 \\ g(N^c y_{max}) N_{pp}^c y_{max} \end{array} \right\} \\ &+ (r - \delta) g(N^c y_{max}) [N_p^c y_{max}]^2 - (r + p^{A*}) N_{pp}^c \bar{y} - 2N_p^c \bar{y} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \Pi^A}{\partial p \partial N} &= \int_0^Q RY dG_{pN} + \int_Q^{N^c y_{max}} [RQ + (r - \delta)(Y - Q)] dG_{pN} \\ &+ [RQ + (r - \delta)(N^c y_{max} - Q)] \left\{ \begin{array}{l} g_p(N^c y_{max}) N_N^c y_{max} \\ + g(N^c y_{max}) N_{pN}^c y_{max} \\ + g_N(N^c y_{max}) N_N^c N_p^c y_{max}^2 \end{array} \right\} \\ &+ (r - \delta) g(N^c y_{max}) N_N^c N_p^c y_{max}^2 \\ &- (r + p) N_{pN}^c \bar{y} - N_N^c \bar{y} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \Pi^A}{\partial p \partial N} &= \int_0^Q RY dG_{NN} + \int_Q^{N^c y_{max}} [RQ + (r - \delta)(Y - Q)] dG_{NN} \\ &+ [RQ + (r - \delta)(N^c y_{max} - Q)] \left\{ \begin{array}{l} g_N(N^c y_{max}) N_N^c y_{max} \\ g_N(N^c y_{max}) (N_N^c y_{max})^2 \\ g(N^c y_{max}) N_{NN}^c y_{max} \end{array} \right\} \\ &+ (r - \delta) g(N^c y_{max}) (N_N^c y_{max})^2 - (r + p^{A*}) N_{NN}^c \bar{y} - m_{NN} \end{aligned}$$

The second order sufficient conditions for the processor's problem with bushel contracts.

$$H^B = \begin{bmatrix} \frac{\partial^2 \Pi^B}{\partial p^2} & \frac{\partial^2 \Pi^B}{\partial p \partial N} & \frac{\partial^2 \Pi^B}{\partial p \partial p^u} \\ \frac{\partial^2 \Pi^B}{\partial N \partial p} & \frac{\partial^2 \Pi^B}{\partial N^2} & \frac{\partial^2 \Pi^B}{\partial p^u \partial N} \\ \frac{\partial^2 \Pi^B}{\partial p \partial p^u} & \frac{\partial^2 \Pi^B}{\partial p^u \partial N} & \frac{\partial^2 \Pi^B}{\partial p^{u2}} \end{bmatrix} \leq 0, \text{ where}$$

$$\begin{aligned} \frac{\partial^2 \Pi^B}{\partial p^2} &= \int_0^Q (R-r) Y dG_{pp} + \int_Q^{N^c y_{max}} (R-r) Q dG_{pp} - p^{B*} N_{pp}^c \bar{y} - 2N_{pp}^c \bar{y} \\ &+ (R-r) Q \left\{ \begin{array}{l} g_p(N^c y_{max}) N_{pp}^c y_{max} \\ g_p(N^c y_{max}) (N_{pp}^c y_{max})^2 \\ g(N^c y_{max}) N_{pp}^c y_{max} \end{array} \right\} \\ &+ p^u \left\{ 2 \int_0^{N^c \bar{y}} N_{pp}^c \bar{y} dG_p + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{pp} + \int_0^{N^c \bar{y}} N_{pp}^c \bar{y} dG + g(N^c \bar{y}) (N_{pp}^c \bar{y})^2 \right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \Pi^B}{\partial N^2} &= \int_0^Q (R-r) Y dG_{NN} + \int_Q^{N^c y_{max}} (R-r) Q dG_{NN} - p^{B*} N_{NN}^c \bar{y} - m_{NN} \\ &+ (R-r) Q \left\{ \begin{array}{l} g_N(N^c y_{max}) N_{NN}^c y_{max} \\ g_N(N^c y_{max}) (N_{NN}^c y_{max})^2 \\ g(N^c y_{max}) N_{NN}^c y_{max} \end{array} \right\} \\ &+ p^u \left\{ 2 \int_0^{N^c \bar{y}} N_{NN}^c \bar{y} dG_N + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{NN} + \int_0^{N^c \bar{y}} N_{NN}^c \bar{y} dG + g(N^c \bar{y}) (N_{NN}^c \bar{y})^2 \right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \Pi^B}{(\partial p^u)^2} &= \int_0^Q (R-r) Y dG_{p^u p^u} + \int_Q^{N^c y_{max}} (R-r) Q dG_{p^u p^u} - p^{B*} N_{p^u p^u}^c \bar{y} \\ &+ (R-r) Q \left\{ \begin{array}{l} g_{p^u}(N^c y_{max}) N_{p^u p^u}^c y_{max} \\ g_{p^u}(N^c y_{max}) (N_{p^u p^u}^c y_{max})^2 \\ g(N^c y_{max}) N_{p^u p^u}^c y_{max} \end{array} \right\} \\ &+ 2 \left\{ \int_0^{N^c \bar{y}} N_{p^u p^u}^c \bar{y} dG + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{p^u} \right\} \\ &+ p^{u*} \left\{ 2 \int_0^{N^c \bar{y}} N_{p^u p^u}^c \bar{y} dG_{p^u} + \int_0^{N^c \bar{y}} N_{p^u p^u}^c \bar{y} dG + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{p^u p^u} + g_{p^u}(N^c \bar{y}) (N_{p^u p^u}^c \bar{y})^2 \right\} \end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \Pi^B}{\partial p \partial N} &= \int_0^Q (R-r) Y dG_{pN} + \int_Q^{N^c y_{max}} (R-r) Q dG_{pN} - p^{B*} N_{pN}^c \bar{y} - N_{N}^c \bar{y} \\
&+ (R-r) Q \left\{ \begin{array}{l} g_p(N^c y_{max}) N_N^c y_{max} \\ g_N(N^c y_{max}) N_N^c N_p^c y_{max}^2 \\ g(N^c y_{max}) N_{pN}^c y_{max} \end{array} \right\} \\
&+ p^{u*} \left\{ \int_0^{N^c \bar{y}} N_N^c \bar{y} dG_p + \int_0^{N^c \bar{y}} N_p^c \bar{y} dG_N + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{pN} + \int_0^{N^c \bar{y}} N_{pN}^c \bar{y} dG + g(N^c \bar{y}) N_N^c N_p^c \bar{y}^2 \right\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \Pi^B}{\partial p^u \partial p} &= \int_0^Q (R-r) Y dG_{p^u p} + \int_Q^{N^c y_{max}} (R-r) Q dG_{p^u p} - p^{B*} N_{p^u p}^c \bar{y} - N_{p^u}^c \bar{y} \\
&+ (R-r) Q \left\{ \begin{array}{l} g_{p^u}(N^c y_{max}) N_{p^u}^c y_{max} \\ g_p(N^c y_{max}) N_{p^u}^c N_p^c y_{max}^2 \\ g(N^c y_{max}) N_{p^u p}^c y_{max} \end{array} \right\} \\
&+ \int_0^{N^c \bar{y}} N_p^c \bar{y} dG + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_p \\
&+ p^{u*} \left\{ \int_0^{N^c \bar{y}} N_{p^u}^c \bar{y} dG_p + \int_0^{N^c \bar{y}} N_p^c \bar{y} dG_{p^u} + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{p^u p} + \int_0^{N^c \bar{y}} N_{p^u p}^c \bar{y} dG + g(N^c \bar{y}) N_{p^u}^c N_p^c \bar{y}^2 \right\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \Pi^B}{\partial p^u \partial N} &= \int_0^Q (R-r) Y dG_{p^u N} + \int_Q^{N^c y_{max}} (R-r) Q dG_{p^u N} - p^{B*} N_{p^u N}^c \bar{y} \\
&+ (R-r) Q \left\{ \begin{array}{l} g_{p^u}(N^c y_{max}) N_N^c y_{max} \\ g_N(N^c y_{max}) N_{p^u}^c N_N^c y_{max}^2 \\ g(N^c y_{max}) N_{p^u N}^c y_{max} \end{array} \right\} \\
&+ \int_0^{N^c \bar{y}} N_N^c \bar{y} dG + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_N \\
&+ p^{u*} \left\{ \int_0^{N^c \bar{y}} N_{p^u}^c \bar{y} dG_N + \int_0^{N^c \bar{y}} N_N^c \bar{y} dG_{p^u} + \int_0^{N^c \bar{y}} (N^c \bar{y} - Y) dG_{p^u N} + \int_0^{N^c \bar{y}} N_{p^u N}^c \bar{y} dG + g(N^c \bar{y}) N_{p^u}^c N_N^c \bar{y}^2 \right\}
\end{aligned}$$

CHAPTER 3. READDRESSING THE FERTILIZER PROBLEM: RECONCILING THE PARADOX

A paper to be submitted to the *American Journal of Agricultural Economics*

Nicholas D. Paulson

Abstract

Pope and Kramer (1979) defined an input to be marginally risk-increasing if the risk-averse agent's marginal risk premium, the wedge between an input's expected marginal product and its price, was positive at the optimum. This implies that *ceteris paribus*, a risk-averse agent will use less of a risk-increasing input than a risk-neutral agent. Empirical work has shown that many inputs to crop production, including fertilizer, are risk-increasing (Roumasset et al. 1989). However, there exists a large body of literature which contends that farmers apply nitrogen in excess of profit-maximizing levels (over-apply) to self protect (Ehrlich and Becker 1972) against fertilizer being a limiting input in years of optimal growing conditions (Babcock 1992; Below and Brandau 2001). This presents somewhat of a paradox with respect to the way in which farmer's use fertilizer under uncertainty. This paper presents a model of optimal input application under output and input uncertainty. Using experimental yield response data, a stochastic production relationship between yield and available soil nitrate is estimated. Input uncertainty is introduced by assuming that available soil nitrate is a stochastic function of fertilizer applied by the farmer. Numerical results imply that while risk-averse farmers may use less fertilizer (i.e. fertilizer is risk-increasing), farmers with both types of risk preference may over-apply nitrogen because of

input uncertainty. In addition to the empirical analysis, primary data from a survey on farmers' risk preferences and subjective beliefs about the relationship between fertilizer and yield variability are presented. The survey data suggest that while farmers do exhibit risk aversion in that certain outcomes are preferred to "gambles", farmers show preference towards yield gambles with small chances of very large gains (high yields) to gambles with small chances of very low yields (crop failures).

Introduction

Two general approaches are used to explore the use of production inputs under uncertainty, often leading to very different conclusions. Pope and Kramer (1979) defined an input to be *marginally risk-increasing (decreasing)*¹ if, under risk aversion, the expected marginal product is greater (less) than the factor price at the expected utility maximizing level (i.e. the marginal risk premium is positive at the optimum). In a *ceterus paribus* framework, this definition can be restated as an input is risk-increasing (decreasing) if the risk averse firm's optimal demand for the input is less (more) than that of the risk neutral firm. A sufficient condition for an input to be risk-increasing is that the variability of output be increasing in the level of input use². Empirical evidence has shown this to be the case for many agricultural inputs, including fertilizer (Roumasset et al. 1989).

Alternatively, an approach which generally relies on the agronomic theory of a limiting input technology shows that the over-application of inputs may be viewed as an

¹ The terms marginally risk-increasing (decreasing) and risk-increasing (decreasing) will be used interchangeably throughout this paper.

² Furthermore, Pope and Kramer (1979) showed that the (then) common multiplicative production function specification implies that all marginally productive inputs are risk-increasing. Moreover, the commonly estimated log-linear production specification is simply a linear representation of the multiplicative form. Thus log-linearity also implies that all marginally productive inputs are risk-increasing.

activity of what Ehrlich and Becker (1972) define as self-protection, or an act to reduce the size of losses. The limiting input theory, first proposed by Von Liebig (1840), implies agricultural production can be defined by a fixed proportions technology where crop yield is determined by the most limiting input. This specification can be characterized by a simple linear response and plateau (LRP) production function (Cate and Nelson 1971).

Complementarity between all inputs is imposed by the LRP specification, so that farmers are thought to use inputs in excess to ensure that none of the elements under their control will be limiting. For example, a farmer may over-apply³ fertilizer to ensure that nutrients are not limiting when exogenous and/or stochastic inputs, such as weather, have optimal realizations (Babcock 1992). As another example, farmers may over-apply inputs to reduce the probability of low yield realizations and increase the probability of high yields. This result implies a distributional effect on yields conditioned on input levels illustrated by Babcock and Hennessy (1996).

The traditional Pope and Kramer definition relies on the global concavity of the producer's utility function. Both risk-averse and risk-neutral farmers value the mean effects of productive inputs equally. However, the risk averse farmer discounts (values) the consequential increase (decrease) in output variability from additional input use. However, because random yield enters linearly into the objective function (profit) of the risk-neutral producer, yield variability has no effect on input choices⁴. Thus, an input which has an

³ For the purposes of this study, over-applying an input is defined as applying more than the profit maximizing level when the input available to the corn plant is a deterministic function of the application rate (i.e. the nitrogen available to the plant is exactly equal to some mean level given the rate of application). If nitrogen fertilizer is applied above this mean optimal rate, then on average nitrogen availability will be in excess of the optimal level. This is consistent with the application rate used by agronomists and crop scientists to formulate application rate recommendations, defined as the "economic optimum" in Sawyer et al. (2006).

⁴ Ignoring the relationship between random output prices and yields.

increasing (decreasing) effect on yield variability will result in lower (higher) levels of use for the risk-averse producer relative to a farmer who is risk-neutral.

Under the limiting input view, farmers may over-apply nitrogen as self-protection (Ehrlich and Becker 1972) even though the theoretical foundations for this argument imply that yield variability is non-decreasing in the level of inputs used⁵. As an example, Babcock (1992) notes that US farmers tend to over-apply fertilizer based on expectations of *ex post* realizations of other uncertain factors despite the fact that yield data implies that yield variability is increasing in the level of fertilizer applied. Further evidence for the limiting input argument was exhibited by Gallagher (1987), where soybean yield variability was shown to be increasing over time. This is attributed to the fact that while yield capacities have increased, the *possibility* of extremely low or zero yield levels may always be present due to forces such as severe weather variability.

Moreover, empirical evidence implies that farmers consistently over-apply fertilizer (SriRamaratnam et al. 1987; Below and Brandau 2001; Sawyer et al. 2006). The National Research Council (1993) estimated that there are up to 8 billion pounds of excess nitrogen left in the soil each year. Yadav, Peterson, and Easter (1997) showed that farmers in southeastern Minnesota applied nitrogen at rates exceeding both recommended rates and estimated profit-maximizing levels. Below and Brandau (2001) state that “Nitrogen fertilizer over-application has been viewed by many as a cheap form of insurance to insure against the possibility of N losses and to make certain that sufficient N is available in case the

⁵ As the level of use for any single input increase from a binding state to a non-binding state the set of possible yield realizations becomes larger, thus increasing the variability of output by definition.

environment is supportive of high yields.”, further justifying the limiting input and self-protection arguments.

This apparent paradox is what is defined as the fertilizer problem. The obvious question is which view is correct? Should the relationship between risk and input use be characterized according to the definition provided by Pope and Kramer (1979), or are inputs used by agricultural producers as self-protection against the forces which are out of their control (i.e. weather, soil characteristics, nutrient availability)? More importantly, is it possible to specify a model which implies that nitrogen is both risk-increasing and “over-applied” by both risk-averse and risk-neutral agents?

This paper attempts to reconcile this paradox by examining these questions using experimental data on corn production in Iowa (Binford, Blackmer, and Cerrato 1992; Blackmer et al. 1989). The experimental nature of the data allows us to examine the effects of nitrogen fertilizer on corn yields *ceterus paribus*. A flexible functional form (translog) is used to model corn yield response to available soil nitrogen after plant emergence in the spring. An estimation procedure proposed by Just and Pope (1979), allowing for flexibility with regards to the effect on input use on output variability, is used to estimate the production function. Consistent with previous empirical studies, it is shown that yield variability is indeed increasing in the amount of soil nitrogen available after emergence in the spring.

Then, following Babcock and Blackmer (1992), available soil nitrogen is specified as a stochastic function of applied nitrogen fertilizer (the farmer’s choice variable). This approach embodies the notion that applied nitrogen fertilizer is not necessarily the critical input; rather the actual nitrogen available to the plant in the soil determines crop yields.

Using numerical techniques, the optimal nitrogen application rates are calculated and

compared for both a risk-neutral and risk-averse producer. While the effects of nitrogen on yield variability do indeed cause the optimal application rates for the risk averse farmer to be lower than those for the risk neutral producer, the effects of uncertainty with respect to available soil nitrogen are shown to be able to increase the optimal application rates above the passive profit-maximizing optimum⁶ for producers with both types of risk preferences (i.e. fertilizer is over-applied). Thus, nitrogen fertilizer would be characterized in this setting as a marginally risk-increasing input according to the Pope and Kramer definition, while it is also “over-applied” relative to the passive optimal application rate due to the uncertainty associated with actual soil nitrogen availability. Both effects are shown to be due to the curvature properties of the (assumed) utility and (estimated) production functions.

Because assumed functional forms may not perfectly represent real-world decision making and estimated relationships are subject to specification error, primary data from actual economic agents (farmers) was also collected by way of a survey instrument. In addition to the analysis of the Blackmer data, results to the survey are provided. The survey, designed following the famous work by Kahneman and Tversky (1979) on prospect theory, shows that the majority of farmers’ preferences over their yield distributions violate some of the classic axioms of expected utility theory. Moreover, the survey results show that while about 50% of the surveyed farmers believe that increased fertilizer use increases yield *variability*, a much smaller proportion of the sample population feel that increased nitrogen use increases *risk*. While farmers do show preference for scenarios with sure payouts to gambles of equal expected value, it is shown that farmers do not necessarily equate yield variability with yield risk as is implicitly assumed in common mean-variance frameworks. In

⁶ The passive profit-maximizing optimal application rate is defined in a later section of the paper.

fact, yield “gambles” with small chances of realizations well above the mean (i.e. bumper crops) are preferred to gambles with small chances of yields well below the mean (i.e. crop failures), when the expected yield levels are equal. This result implies that risk-averse farmers may not consider large yield realizations as risky outcomes. However, a globally concave utility function discounts yield variability above and below the mean equally.

The remainder of this paper is organized as follows. The next section provides some background on previous work related to this study. The third section specifies the yield response model. The fourth section discusses the data and outlines the estimation techniques employed. Section five reports and discusses the estimation results. Section six provides numerical examples of optimal nitrogen application rates under various circumstances. Section seven outlines the farmer survey and the collection methods, as well as a detailed look into the survey results. The final section provides some concluding remarks and discusses areas for further research.

Literature Review

The literature on the relationship between risk, production levels, and input use in agriculture is vast. Ratti and Ullah (1976) examined the effects of production uncertainty on input use in a two factor model. They showed that input demands under uncertainty are less for the risk averse firm, compared to the risk neutral firm, given assumptions on the elasticities of the marginal product curves and the level of complementarity between the two factors. Furthermore, they show that the required assumptions generally hold for many of the (then) common production function specifications, including the Cobb-Douglass, CES, and Transcendental models. MacMinn and Holtmann (1983) analyze input choice under a very

general form of technological uncertainty, finding that risk-averse agents may increase or decrease input use as the level of production uncertainty increases.

Rothschild and Stiglitz (1970 and 1971) showed that the effect of increases in risk, defined by a mean preserving spread, on optimal input levels depends on the curvature of the marginal utility or product curve with respect to the stochastic shock⁷. The effects of price uncertainty have also been explored extensively (Sandmo 1971; Ishii 1977; Hartman 1976).

Examples of agricultural inputs with both risk-increasing and risk-decreasing characteristics have been reported in the literature. Pest control inputs have been shown to provide some level of protection against production uncertainty implying a risk-decreasing effect (Feder 1979). However, Hurley and Babcock (2003) find that pesticides would be defined as risk-increasing by the Pope and Kramer definition. As another example, increases in the capital-labor ratio have been shown to reduce the effects of weather variability on production in agriculture (see Pope and Kramer 1979 for examples).

Hurley, Mitchell, and Rice (2004) emphasize the endogeneity of risk caused by input choices. They apply their conceptual model of the adoption of Bt corn hybrid technology to two Midwestern counties and find that Bt corn, while generally thought to be an input used as self-protection against corn borer infestation, may be defined as a risk-increasing or risk-decreasing input depending on the price of Bt seed. However, their model implies an interesting result in that “When planting corn is optimal, Bt corn is risk increasing if the expected loss it eliminates exceeds its price...” (Proposition 1, p. 347 of Hurley, Mitchell, and Rice 2004). In other words, Bt corn is risk increasing when the expected benefit of its

⁷ Rothschild and Stiglitz (1971) assumed risk-aversion for the optimizing agent. For a risk-neutral agent under production uncertainty, their curvature result applies to the marginal product curve.

adoption is greater than the cost of adoption (i.e. when Bt corn is expected to increase expected profits).

SriRamaratnam et al. (1987) used an experimental approach to elicit farmer's subjective beliefs about the responsiveness of sorghum yields to nitrogen fertilizer in Texas. They found that farmers generally overestimate the response of yields to fertilizer causing them to over-apply. More recent studies mentioned in the previous section have also found evidence that farmers apply fertilizer at rates which exceed the profit-maximizing level (NRC 1993; Yadav, Peterson, and Easter 1997; Below and Brandau 2001).

Just and Pope (1979) use a three-stage estimation process to separate the effects of nitrogen fertilizer on average yield and yield variability. They find that both corn and oat yield variability are indeed increasing functions of nitrogen fertilizer application rates. Additionally, Just and Pope (1979) provide references to previous studies in which yield variability was found to be increasing in fertilizer application rates, although the goal of their paper was to show that the multiplicative, or log-linear, forms used in the estimation of these studies imposed this condition. More recently, Ramaswami (1992) provided more generalized conditions under which input could be characterized according to the Pope and Kramer definition⁸. Using the same experimental data, Ramaswami (1992) showed that, using his relaxed definition, nitrogen fertilizer was risk-increasing at low (high) application rates for cotton (corn). However, assumptions on preferences were required to sign the marginal risk premium for high (low) application rates for cotton (corn).

⁸ Ramaswami (1992) derived conditions under which an input could be characterized as risk increasing (decreasing) based purely on technological assumptions, noting that Pope and Kramer's original definition includes assumptions on both preferences and technology. His result yields the weakest condition necessary to define an input as risk increasing (decreasing).

Paris and Knapp (1989) outlined multiple methods for estimation of LRP functions from input and output data. Using these methods with experimental data on corn response to nitrogen and phosphorus fertilizers, Paris (1992) shows that the LRP specification provided the best interpretation of the production relationship. Lanzer and Paris (1981) estimated LRP production functions for wheat and soybean production in Brazil. By incorporating nutrient carry-over, they demonstrate the inefficiency of fertilizer application rate recommendations which are based on the traditional polynomial fertilizer response functions. Examples of other applications of the LRP production function to agricultural studies include Babcock and Blackmer (1992, 1994) and Babcock (1992). Babcock and Blackmer (1992) use the LRP specification to model corn yield response as a function of available soil nitrogen. They find that nitrogen application rates could be reduced by up to 38% through the use of late spring soil tests followed by side-dressed nitrogen applications if the soil tests indicate low levels of available soil nitrogen. Their results imply that farmers may be over-applying nitrogen to offset the uncertainty with respect to the nitrogen that will be available to the plants during growth. In another study, Babcock and Blackmer (1994) use the LRP specification with experimental data on Iowa corn production to show that there may be a positive relationship between optimal nitrogen fertilizer application rates and growing conditions, indicating that the notion of over-applying fertilizer inputs to take advantage of the “good years” may have some validity.

In another application of the LRP function to agricultural data, Babcock (1992) shows that uncertainty induces the over-application of nitrogen fertilizer as long as the slope of the LRP function, in the range where nitrogen is limiting, is more than double the price of nitrogen fertilizer. While this result relies on a distributional assumption, it is quite intuitive.

Given the expected value for “growing conditions”, the LRP specification implies an optimal application rate equal to the point where nitrogen fertilizer becomes non-binding (i.e. the kink in the LRP function where yield response reaches the plateau)⁹. When uncertainty with respect to growing conditions is introduced there is a 50% chance of either a better or worse realization for growing conditions. Thus, there is a 50% chance that nitrogen could become limiting, which implies Babcock’s result.

However, the LRP approach to modeling production in agriculture has not been adopted extensively in the literature because economists prefer to work with smooth differentiable functions (Lanzer and Paris 1981). Moreover, the notion of a limiting input is based on soil science theory at the plant level, while economists tend to examine problems from a more aggregated viewpoint at a field or whole farm level. Berck and Helfand (1990) reconciled the opposing views of the LRP and differentiable polynomial production function specifications by showing that the traditional functional forms can be derived as an aggregation of the LRP model across many heterogeneous inputs. The decision to estimate a smooth, differentiable translog production function for this study rather than using the LRP model was made based on their results.

Model

Consider a risk-neutral farmer who chooses an amount of fertilizer to apply x , at unit price w , to produce stochastic output \tilde{q} to maximize expected profits $E[\pi]$. Furthermore, assume that the amount of fertilizer relevant to production is that which is available in the

⁹ This is assuming the slope of the LRP function over the range where nitrogen would be the binding input is less than the unit price of nitrogen fertilizer. If the slope, or the marginal response, is less than the unit price the optimal application rate is zero.

soil \tilde{x} , which is assumed to be a stochastic function of applied fertilizer. The relationship between applied fertilizer and available fertilizer is given by

$$(1.1) \quad \tilde{x} = \underline{x} + x + \phi\mu, \text{ where } \mu = \tilde{x} - E[\tilde{x}] \text{ and } \tilde{x}, \phi \geq 0.$$

The error term μ is mean zero by construction so that the expected level of available nutrients in the soil is equal to the amount in the soil prior to fertilizer application \underline{x} ¹⁰ plus the amount of fertilizer applied x . The special case where $\phi = 0$ is that of input certainty with respect to applied fertilizer. Increasing the value of ϕ is a measure of increasing input uncertainty by way of a mean preserving spread. This is consistent with the definitions of increasing risk proposed by Rothschild and Stiglitz (1970) in that $\phi > 0$ implies available fertilizer is equal to applied fertilizer “plus noise”. Available fertilizer may differ from what is applied for a variety of reasons such as nutrient losses (or gains) from weather events¹¹ and differences in natural levels of nutrients due to heterogeneity with respect to soil types and previous practices (i.e. crop rotations), as well as uncertainty with respect to application technologies.

Output is assumed to be an increasing concave function in the amount of available fertilizer \tilde{x} , $\tilde{q}_{\tilde{x}} \geq 0, \tilde{q}_{\tilde{x}\tilde{x}} \leq 0$ for $\forall \tilde{x} \geq 0$. All other production inputs, denoted by vector \mathbf{z} with unit price vector \mathbf{r} , are taken as given. Thus, the analysis is limited to the maximization of expected profits conditional on all inputs (other than fertilizer) being exogenous.

$$(1.2) \quad \pi = \tilde{q}(\tilde{x}(x, \phi), \tilde{\varepsilon}; \mathbf{z}) - wx - \mathbf{r}'\mathbf{z}.$$

¹⁰ The value of \underline{x} will, in general, be a function of a number of variable including the previous year's crop (rotation effects) and the specific soil type.

¹¹ Farmers in the Midwest generally apply nitrogen fertilizer for corn production in the fall after harvest or in the spring prior to planting. In either case there is generally a period of time between application and when nutrient uptake by the plant occurs. Therefore, (stochastic) weather events such as rainfall that occur between application and plant growth may affect available nutrient levels in the soil.

The stochastic component of output $\tilde{\varepsilon}$ is assumed to be a mean-zero disturbance whose variance may or may not depend on inputs \tilde{x} , x , and \mathbf{z} . Without loss of generality, it is also assumed that high draws of $\tilde{\varepsilon}$ are associated with high draws of output \tilde{q} . The stochastic disturbance could be thought of as a proxy for growing conditions throughout the production period, where “better” growing conditions are given by larger draws of $\tilde{\varepsilon}$. The farmer’s profit maximization problem is given below, where it is assumed that all inputs must be non-negative.

$$(1.3) \quad \max_x E[\pi | \mathbf{z}] = E[\tilde{q}(\tilde{x}(x, \phi), \tilde{\varepsilon}; \mathbf{z})] - wx - \mathbf{r}'\mathbf{z}$$

subject to $x \geq 0$ and equation (1.1)

$$(1.4) \quad E[\pi_x] = E[\tilde{q}_{\tilde{x}}(\tilde{x}(x^*), \tilde{\varepsilon}; \mathbf{z})] - w \leq 0, x^* \geq 0$$

Denoting the expected profit-maximizing level of applied fertilizer by $x^*(w, \phi)$, the first order condition for a maximum is given by equation (1.4), where derivatives are denoted with a subscript and the first order condition holds with equality if $\hat{x}^* > 0$. The second order condition requires that the expected value of the second derivative of output with respect to fertilizer be non-positive $E[\tilde{q}_{\tilde{x}\tilde{x}}(\tilde{x}(x^*), \tilde{\varepsilon})] \leq 0$, which is satisfied given the curvature assumptions on \tilde{q} . The optimal application rate given in (1.4) when $\phi = 0$ will be defined as the passive profit-maximizing optimum because it ignores the uncertainty associated with soil nutrient levels. This leads to the following definition

Definition 1: An input is said to be over(under) applied if the application rate is greater (less) than the passive risk-neutral optimum $x^(w, \phi = 0)$.*

Now consider a risk-averse farmer who chooses the amount of fertilizer to apply to maximize the expected utility of profits given in (1.2). The risk-averse farmer's problem is given by

$$(1.5) \quad \max_x E[U(\pi | \mathbf{z})] = E[U(\tilde{q}(\tilde{x}(x, \phi), \tilde{\varepsilon}; \mathbf{z}) - wx - \mathbf{r}'\mathbf{z})]$$

subject to $x \geq 0$ and equation (1.1)

where the utility function U is assumed to be increasing and concave, $U_\pi \geq 0, U_{\pi\pi} \leq 0$ for $\forall \pi$. Let the expected utility maximizing level of fertilizer be denoted by $x^{**}(w, \phi)$. The

risk-averse farmer's first and second order conditions are given by

$$(1.6) \quad E[U_x] = E[U_\pi (\tilde{q}_{\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z}) - w)] \leq 0, x^{**} \geq 0 \text{ and}$$

$$(1.7) \quad E[U_{xx}] = E[U_{\pi\pi} (\tilde{q}_{\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z}) - w)^2 + U_\pi \tilde{q}_{\tilde{x}\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z})] \leq 0.$$

Note the second order condition in (1.7) is satisfied given the curvature assumptions on U and \tilde{q} .

Suppose that interior solutions exist for both the risk-neutral and risk averse farmer so that (1.4) and (1.6) both hold with equality. Then, using $Cov(x, y)$ to denote the covariance between x and y , (1.6) can be rewritten as

$$(1.8) \quad E[\tilde{q}_{\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z})] - w = -\frac{Cov(U_\pi, \tilde{q}_{\tilde{x}})}{E[U_\pi]},$$

where $-\frac{Cov(U_\pi, \tilde{q}_{\tilde{x}})}{E[U_\pi]}$ is what Pope and Kramer (1979) defined as the marginal risk premium

(MRP). Pope and Kramer (1979) defined the input x to be risk increasing (decreasing) if the

MRP is positive (negative). Or, $x^{**} \leq (>)x^*$ if the MRP is positive (negative) at the

optimum. Given positive marginal utility, the denominator is positive so that the sign of the MRP is the opposite of the sign of the covariance between marginal utility and marginal product.

Differentiating the first order conditions (1.4) and (1.6) with respect to ϕ yields the following comparative static results.

$$(1.9) \quad \frac{\partial x^*}{\partial \phi} = -\frac{\text{Cov}(q_{\tilde{x}\tilde{x}}, \mu)}{E[q_{\tilde{x}\tilde{x}}]}$$

$$(1.10) \quad \frac{\partial x^{**}}{\partial \phi} = -\frac{\text{Cov}\left(U_{\pi\pi} \left[\tilde{q}_{\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z}) - w \right], \mu\right)}{E\left[U_{\pi\pi} \left(\tilde{q}_{\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z}) - w \right)^2 + U_{\pi} \tilde{q}_{\tilde{x}\tilde{x}}(\tilde{x}(x^{**}), \tilde{\varepsilon}; \mathbf{z})\right]}$$

The signs of (1.9) and (1.10) are equal to the signs of the numerators, because the denominator of each term is negative by the second order conditions of the risk-neutral and risk-averse farmer's maximization problems, respectively. The sign of equation (1.9) depends on the sign of $q_{\tilde{x}\tilde{x}}$, with $\frac{\partial x^*}{\partial \phi} > (<) 0$ as $q_{\tilde{x}\tilde{x}} > (<) 0$. Similarly, a sufficient condition for $\frac{\partial x^{**}}{\partial \phi} > (<) 0$ is that $q_{\tilde{x}\tilde{x}}$ and $U_{\pi\pi} > (<) 0$. These are analogous to the results shown by Rothschild and Stiglitz (1971).

Assuming convex (concave) marginal utility and convex (concave) marginal product implies that the optimal level of input use when $\phi > 0$ is greater (less) than when $\phi = 0$.

However, one of the purposes of this analysis is to compare the risk-averse farmer's optimal input use under input uncertainty $x^{**}(w, \phi > 0)$ to the risk-neutral farmer's optimal fertilizer application without input uncertainty $x^*(w, \phi = 0)$ (i.e. the passive optimum). To

accomplish this task experimental data on yield response to nitrogen is used to estimate a yield response function for corn.

Data and Estimation

Corn production or yield, q , is modeled as a function of soil nitrogen x and a stochastic error term ε . Following Just and Pope (1979), the production function is the sum of a mean and variance component which are both functions of the input x . The error term enters multiplicatively with the variance component and is assumed to be distributed according to the standard normal distribution.

$$(1.11) \quad q = f(x) + h^{\frac{1}{2}}(x)\varepsilon, \quad \varepsilon \sim N(0,1)$$

This of course implies that the variance of output is equal to $h(x)$ and the effect of input use on output variability is given by $\frac{\partial \text{Var}(q)}{\partial x} = h_x$. The error term in the production function is assumed to capture the effects of exogenous forces, such as weather, while the scale of this variability is determined endogenously by input levels through the variance function component. Thus, this specification captures the endogeneity of risk emphasized by Hurley, Mitchell, and Rice (2004), where risk is loosely defined as yield variability around the mean. Furthermore, the marginal productivity of an input, captured by f , does not impose any *a priori* restrictions on that input's effect on yield variability. The effect of input levels on yield variability depends on the sign of h_x ¹².

¹² Note that $h_x > 0$ is a sufficient condition for the $\text{MRP} \geq 0$ assuming concave utility (Pope and Kramer 1979; Ramaswami 1992).

The data used for this study comes from the earlier works of Binford, Blackmer, and Cerrato (1992) and Blackmer et al. (1989). Subsets of the data have also been used by Babcock and Hennessy (1996) and Babcock and Blackmer (1994, 1992). The data set contains information on corn yields, nitrogen fertilizer application rates, and results from a late spring soil nitrogen test for 15 experiment stations across the state of Iowa collected from 1985 to 1990. A significant amount of weather variability is included in the data with years of excellent growing conditions and high yield levels (1987, 1990) and years of extremely poor growing conditions and low yield levels (1988 drought). All input levels, other than applied nitrogen fertilizer, were held constant and at non-limiting levels across years and sites to isolate the effects of nitrogen fertilizer on corn yields. Additionally, both continuous corn (corn-corn) and corn following soybeans (corn-soybean) rotations were examined.

The corn-soybean rotation data consisted of a total of 750 observations, while the corn-corn data included 1248 observations. Data on continuous corn covered all 6 years and all 15 experiment station locations in the full data set. Data for the corn-soybean rotation was only available for 8 experiment station sites over a 4 year period (1987-1990). Nitrogen fertilizer rates ranged from zero to 300 pounds per acre of nitrogen fertilizer in 25-50 lb. increments, with three repetitions of each application rate performed annually at each experiment station site. A late spring soil nitrate test was also conducted and recorded to determine the level of nitrogen, in parts per million (ppm), available in the soil for plant growth.

The available soil nitrate levels were highly (but not perfectly) correlated with fertilizer application rates (0.70 correlation coefficient). Soil nitrate levels ranged from 3.8 to

134.6 ppm, with an average of 27.5 (30.7) ppm and standard deviation of 18.1 (16.3) ppm in the corn-corn (corn-soybean) data. Yields ranged from 4 to 218 bushels per acre, with an average of 118.4 (143.2) bushels per acre with a standard deviation of 45.9 (39.7) bushels per acre in the corn-corn (corn-soybean) data. Overall, average soil nitrate and yield levels were higher and less variable for the corn-soybean rotation data which is consistent with previous findings regarding the benefits of crop rotation.

Using a three stage approach outlined by Just and Pope (1979), translog functional forms were estimated for the mean and variance components in the production function. The translog form was compared to alternative specifications, including the Cobb-Douglas, linear, and quadratic specifications. A likelihood ratio test rejected the Cobb-Douglas form, while the linear and quadratic specifications were found to provide an inferior fit to the translog form when plotted against the data. While basic soil characteristics may have differed between experiment sites, it is noted in Blackmer et al. (1989) and Binford, Blackmer, and Cerrato (1992) that much care was taken to make each of the observations as comparable to each other as possible. Tillage, planting, and harvest practices were coordinated to be nearly identical across the experiment sites with regard to both methods and timing. Of course, heterogeneity due to weather variability and site specifics such as soil type were not able to be controlled. Dummy variables were included to capture site (d_s) effects^{13,14}.

¹³ In an earlier version of this paper dummies were also included to capture year effects. However, this had the effect of controlling for some of the exogenous risk farmers face when they choose input levels. Therefore, the year dummies were removed. Site dummies were left in the analysis, assuming site effects represent farm-specific measures that would be known by the producer. Thanks to Sergio Lence and Bruce Babcock of Iowa State University for their comments on this issue.

¹⁴ Just and Pope (1979) use a variance components technique to capture time and site effects in their experimental data. They note that Maddala, and Wallace and Hussein discuss the advantages to variance

The first stage of the estimation procedure provides consistent estimates of the mean yield component parameters by estimating the following equation using a non-linear least squares (NLS) estimator¹⁵.

$$(1.12) \quad q_{i,s} = f(x_{i,s}) + \varepsilon_{i,s}^* = \alpha_0 x_{i,s}^{\alpha_x} \exp\left(\frac{1}{2} \alpha_{xx} \log(x_{i,s})^2\right) + \alpha_s' d_s + \varepsilon_{i,s}^*,$$

$$\text{where } \varepsilon_{i,s}^* = h^{\frac{1}{2}}(x_{i,s}) \varepsilon_{i,s}$$

$$E[\varepsilon_{i,s}] = E[\varepsilon_{i,s} | x] = 0$$

Given the assumptions on ε , the composed error term ε^* is normally distributed with a zero mean, but is heteroskedastic. Thus while the first-stage parameter estimates are unbiased and consistent, they are not efficient. Moreover the standard errors cannot be used for hypothesis testing due to the heteroskedastic nature of the error term (Greene 2003).

Noting that $E[(\varepsilon_{i,s}^*)^2] = E[h(x_{i,s}) \varepsilon_{i,s}^2] = h(x_{i,s})$ and $(\varepsilon_{i,s}^*)^2 = E[(\varepsilon_{i,s}^*)^2] \xi_i$, where ξ_i is such that $E[\xi_i] = 1$, the consistent parameter estimates for the mean component from the first stage can be used to obtain consistent estimates of the first stage residuals, $\hat{\varepsilon}_{i,s}^*$. The squared residuals can then be regressed, in a non-linear framework, on the nitrogen levels in a translog functional form to obtain consistent estimates of the parameters for the variance component of the production function.

components versus the dummy variable approach. Wallace and Hussein note that asymptotic efficiency is only attained through the variance components approach with data where regressors repeat from year to year, which is the case with the applied nitrogen rates. However, we adopt available soil nitrogen as an independent variable, which is continuous. Thus, dummy variables are assumed to be adequate to capture site effects.

¹⁵ The non-linear least squares estimation was carried out in MatLab using code written by the author. The model was estimated using the linearized approach and convergence criterion discussed in Greene (2003).

Finally, the third stage of estimation is done within a generalized NLS estimation procedure, where the covariance matrix is estimated using the second stage parameter estimates for the variance component. The third stage of the procedure re-estimates the parameters of the mean yield component, providing unbiased, consistent, and efficient parameter estimates. The use of dummy variables in the model implies a simple heteroskedastic covariance structure where the non-diagonal elements of the estimated covariance matrix are equal to zero. Production functions for the corn-corn and corn-soybean rotations were estimated to separate the rotational effects. The estimation results are reported in the next section.

Estimation Results

Table 1 reports the parameter estimates (t-statistics) for the corn-soybean rotation data. Table 2 reports the parameter estimates (t-statistics) for the continuous corn data. The first and third set of columns in each table report the first-stage and third-stage parameter estimates, respectively, for the mean component of the production function. The second set of columns in the tables report the coefficient estimates for the variance component of the production function.

For each rotation, an unrestricted model was estimated including a full set of site dummies. A restricted model was then estimated, eliminating the site dummies which were not statistically significant at a 5% significance level in the unrestricted model. Comparisons of yield plots across years and sites were consistent with the statistical significance of the dummy estimates. Site 9 was arbitrarily chosen as the baseline site. Negative effects for site

13 were shown for corn following soybeans, while negative effects at sites 13 and 4 were significant for the continuous corn data¹⁶.

Table 1. Production Function Estimates, Corn-Soybean Rotation

	1 st Stage		2 nd Stage		3 rd Stage	
	Unrestricted	Restricted	Unrestricted	Restricted	Unrestricted	Restricted
α_0	56.89 (0.30)	62.16 (0.26)	164.46 (0.002)	71.19 (0.003)	63.01 (0.28)	67.92 (0.24)
α_x	0.522 (23.71)	0.482 (19.90)	0.779 (0.578)	1.465 (0.902)	0.458 (21.43)	0.426 (17.94)
α_{xx}	-0.145 (-69.23)	-0.130 (-56.35)	-0.115 (-1.003)	-0.338 (-2.412)	-0.126 (-61.91)	-0.112 (-49.79)
<i>Site dummies</i>						
d_3	9.93 (0.445)				12.40 (0.546)	
d_{10}	31.07 (1.282)				30.07 (1.247)	
d_{11}	-7.23 (-0.391)				-8.02 (-0.436)	
d_{12}	-7.27 (-0.393)				-6.50 (-0.349)	
d_{13}	-31.80 (-1.689)	-38.01 (-2.275)			-31.51 (-1.669)	-38.22 (-2.288)
d_{16}	43.55 (0.777)				43.74 (0.779)	
d_{17}	-3.94 (-0.081)				-1.83 (-0.037)	

Average yield levels were found to be increasing and concave functions of available soil nitrate for both rotations. Yield variability was also found to be an increasing function of available soil nitrate for both rotations. Figures 1 plots the yield and variance functions for continuous corn and corn-soybean rotations. The yield functions for soil nitrate tend to flatten out as the level of soil nitrate increases beyond 60 ppm. For the continuous corn data, yield variability increases almost linearly with soil nitrate (above 10 ppm), while yield variability

¹⁶ The estimation results should not be interpreted as any type of best-fit yield response function for fertilizer recommendations. This was not the goal of the analysis. Moreover, the experimental data is up to twenty years old and is not expected to reflect yield response to nitrogen fertilizer for more modern hybrid genetics.

is estimated to be increasing but concave in soil nitrate. The estimation results confirm that yield variability does indeed increase with the level of available soil nitrate (i.e. $h_x > 0$) for both crop rotations. Therefore, by the Pope and Kramer definition, nitrogen would be considered a risk-increasing input to corn production. The next section examines the effects on input on uncertainty on farmers' optimal application rates for nitrogen fertilizer.

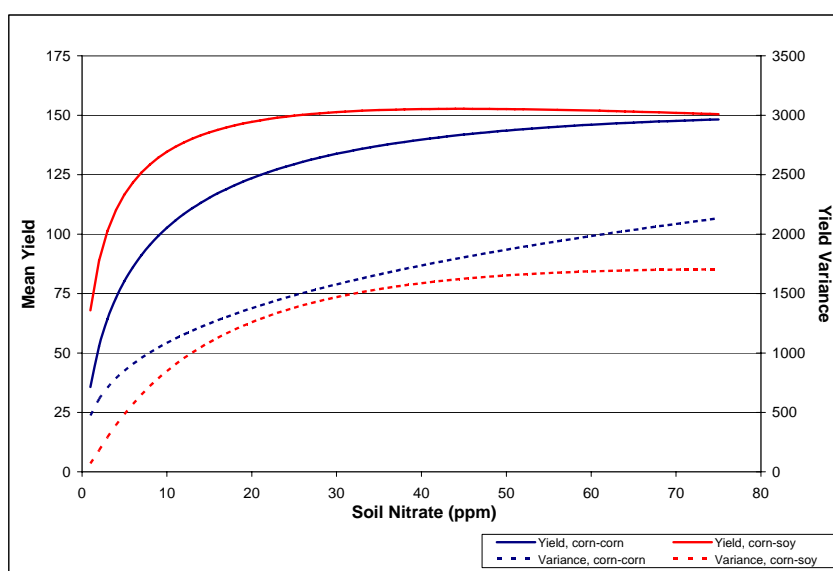


Figure1. Estimated yield and variance functions, corn-corn and corn-soy rotations

Table 2. Production Function Estimates, Corn-Corn Rotation

	1 st Stage		2 nd Stage		3 rd Stage	
	Unrestricted	Restricted	Unrestricted	Restricted	Unrestricted	Restricted
α_0	26.07 (1.094)	33.39 (0.953)	584.85 (0.003)	474.30 (0.004)	27.29 (1.071)	35.71 (0.911)
α_x	0.776 (61.85)	0.651 (53.28)	0.175 (0.747)	0.372 (1.668)	0.750 (61.16)	0.606 (50.66)
α_{xx}	-0.178 (-150.64)	-0.142 (-123.07)	0.050 (2.41)	-0.011 (-0.554)	-0.172 (-148.25)	-0.128 (-113.10)
<i>Site dummies</i>						
d_1	-13.31 (-0.598)				-12.66 (-0.565)	
d_3	11.82 (0.551)				15.25 (0.689)	
d_4	-96.51 (-2.122)	-100.71 (-2.051)			-94.18 (-2.063)	-98.68 (-2.009)

Table 2. Production Function Estimates, Corn-Corn Rotation (continued)

	1 st Stage		2 nd Stage		3 rd Stage	
	Unrestricted	Restricted	Unrestricted	Restricted	Unrestricted	Restricted
d ₅	22.22 (0.767)				21.34 (0.738)	
d ₆	11.45 (0.440)				8.52 (0.333)	
d ₈	2.41 (0.097)				1.39 (0.056)	
d ₁₀	-1.90 (-0.094)				-1.33 (-0.065)	
d ₁₁	10.92 (0.484)				8.80 (0.395)	
d ₁₂	-7.10 (-0.365)				-4.81 (-0.243)	
d ₁₃	-50.36 (-2.788)	-54.63 (-3.255)			-47.74 (-2.604)	-52.11 (-3.097)
d ₁₄	-8.06 (-0.142)				-4.89 (-0.085)	
d ₁₅	-5.76 (-0.101)				-5.60 (-0.098)	
d ₁₆	50.79 (0.790)				52.03 (0.804)	
d ₁₇	-0.809 (-0.015)				0.051 (0.0009)	

Optimal Fertilizer Rates

For any fertilizer application rate, the level of available soil nitrate in late spring is stochastic. Using the same experimental data, Babcock and Blackmer (1992) estimated three-parameter gamma distributions for the distribution of available soil nitrate \tilde{x} conditional on fertilizer application rates. They specified the gamma distribution's parameters as linear functions of the application rates x .

$$(1.13) \quad p(\tilde{x}) = \frac{(\tilde{x} - \gamma)^{\theta-1} \exp[-(\tilde{x} - \gamma)/\lambda]}{\lambda^{\theta} \Gamma(\theta)}$$

where $\theta > 0, \lambda > 0, \gamma > 0$

$$\theta = \theta_0 + \theta_1 x$$

$$\lambda = \lambda_0 + \lambda_1 x$$

$$\gamma = \gamma_0 + \gamma_1 x$$

Maximum likelihood estimates for the parameter functions were computed to confirm and replicate the results of Babcock and Blackmer (1992). Alternative specifications were also examined, but likelihood ratio tests showed that their linear specifications best fit the experimental data. The parameter estimates (t-statistics) are equivalent to those reported in Babcock and Blackmer (2002) (Table 1) and are reported in table 3. As in their study, the value of γ_0 was restricted to equal to zero, implying a lower bound of zero ppm for soil nitrate concentration when no fertilizer is applied. Both the mean and variance of soil nitrate levels are increasing in the level of fertilizer applied for both rotations. The skewness exhibited in the soil nitrate distributions also increases with the amount of nitrogen fertilizer applied. See Babcock and Blackmer (1992) for plots of the distributions at varying nitrogen fertilizer application rates.

Table 3. Parameter Estimates (t-statistics) for Soil Nitrate Distributions

	Rotation	
	Continuous Corn	Corn-Soybean
θ_0	4.92 (16.4)	5.94 (10.1)
θ_1	-0.00478 (4.78)	-0.00468 (2.23)
λ_0	1.963 (14.1)	2.178 (11.6)
λ_1	0.0279 (17.4)	0.0167 (10.4)
γ_1	0.0366 (5.7)	0.0657 (7.9)

Using the soil nitrate distribution parameter estimates, soil nitrate draws were generated over a grid of nitrogen application rates. For each soil nitrate draw, expected profit

and utility were computed assuming a constant absolute risk aversion (CARA) utility function for the risk-averse farmer and using a random draw of standard normal deviates for ε with the estimated production function¹⁷. This defined distributions of expected profits and utilities for each nitrogen application rate.

$$(1.14) \quad U(\pi) = -\exp[-\beta\pi]$$

The nitrogen application rates which maximized expected profits and expected utility were found for both rotation types and over a range of risk aversion levels. The relative price of nitrogen fertilizer was set to calibrate optimal application rates close to current recommendations under risk-neutrality and certainty with respect to soil nitrate levels for both continuous corn and corn-soybean rotations¹⁸. These results were compared to the expected profit and utility maximizing application rates under certainty with respect to soil nitrate availability¹⁹.

Following Babcock, Choi, and Feinerman (1993) the coefficient of absolute risk aversion was calibrated to 0.017 to yield a risk premium ratio of 25% for both the corn-soybean and continuous corn rotations. Risk aversion levels above and below this calibrated level were examined. The effect of uncertainty with respect to soil nitrate levels on optimal fertilization rates will depend on the curvature implied by the production and utility functions (Rothschild and Stiglitz 1971). The estimated production function implies convex marginal product, although no *a priori* restrictions were made to ensure this. Additionally, CARA utility functions belong to the family of non-decreasing absolute risk aversion functions,

¹⁷ For each draw of soil nitrate, the yield distribution is determined by the variance component of the yield function and the draw for the exogenous risk component, ε .

¹⁸ Current recommendations for application rates in southern Minnesota are around 130 (170) lbs/acre for corn following soybeans (continuous corn).

¹⁹ For the case of input certainty, the level of soil nitrate was set equal to the mean of the gamma distribution implied by the fertilizer application rate.

implying convex marginal utility. Based on equations (1.9) and (1.10), the optimal nitrogen application rates were expected to be larger under soil nitrate concentration uncertainty.

Table 4 reports the optimal nitrogen application rates over varying risk attitudes for the corn-soybean rotation. The third column reports optimal application rates when soil nitrate is a deterministic function of applied nitrogen ($\phi = 0$). The fourth column reports optimal application rates when soil nitrate is a stochastic variable distributed according to the gamma distribution conditional on the fertilizer application rate according to relation (1.13) and the parameter estimates reported in table 3. The input uncertainty cases reported in the fourth column correspond to $\phi > 0$ ²⁰. For all levels of risk aversion the optimal application rate for the risk averse farmer \hat{x}^{**} is less than that of the risk-neutral farmer \hat{x}^* . These results apply for both cases of input certainty ($\phi = 0$) and uncertainty ($\phi > 0$), and illustrate the fact that nitrogen fertilizer is risk-increasing.

However, comparing the optimal application rates under input certainty and input uncertainty shows that farmers with all risk preferences apply more fertilizer when soil nitrate is stochastic ($\phi > 0$). Optimal application rates under input uncertainty are 8-17% greater than when soil nitrate is certain given applied nitrogen fertilizer. Note also that for farmers with risk aversion coefficients less than 0.001, the optimal application rate under input uncertainty is greater than the optimal rate for the risk-neutral farmer when soil nitrate is certain (the passive optimum). Given uncertainty with respect to soil nitrate concentrations, some risk-averse farmers may optimally over-apply nitrogen fertilizer. The

²⁰ The fourth column in tables 4 and 5 report optimal application rates when $\phi > 0$. However, each row in tables 4 and 5 represent different values of ϕ conditional on the amount of fertilizer applied by the relation (1.13) and the parameter estimates in table 3.

result that a farmer would over-apply nitrogen fertilizer under soil nitrate uncertainty depends on the curvature of the yield response and utility functions (Rothschild and Stiglitz 1971). A convex marginal product curve implies that, on average, the gains from an additional unit of fertilizer are greater than the losses or the price of the additional unit, which is the intuition behind the result of Rothschild and Stiglitz (1971).

Table 4. Optimal Nitrogen Application Rates (lbs/acre), Corn-Soybean Rotation

	β	$\phi = 0$	$\phi > 0$
x^*	Risk Neutrality	105	113
	0.0001	105	113
	0.001	104	112
x^{**}	0.01	88	98
	0.02	70	82

The optimal nitrogen application rates for continuous corn are reported in Table 5. Again, as the level of risk aversion increases the optimal application rates decline because fertilizer is a risk-increasing input. However, application rates for farmers with all types of risk preferences increase by 2-6% when input uncertainty is introduced. As with the corn-soybean rotation results, farmers with risk aversion coefficients less than 0.001 have optimal application rates greater than risk-neutral farmers under input certainty.

Table 5. Optimal Nitrogen Application Rates (lbs/acre), Corn-Corn Rotation

	β	$\phi = 0$	$\phi > 0$
x^*	Risk Neutrality	169	172
	0.0001	169	172
	0.001	167	170
x^{**}	0.01	153	159
	0.02	138	146

Farmer Survey Results

In addition to the empirical analysis of the Blackmer nitrogen response data, a survey was distributed to Midwestern corn farmers to analyze their risk preferences and subjective beliefs about the relationship between fertilizer use and yield risk. The results thus far have been based on assumed preference relationships and an estimated production function. The collection of primary data, by way of the survey, was done in an attempt to gain additional insight into whether the assumptions made on farmer preferences are consistent with the actions and beliefs of actual farmers. The survey results were intended to be used to try and validate or discredit the empirical and theoretical findings of this and previous studies.

The survey was distributed to farmers in Illinois, Iowa, Missouri, Minnesota and North Dakota during August and September of 2006. The majority of the surveys were completed voluntarily by corn producers who attended informational meetings held by Decision Commodities²¹ of Ames, IA. Additionally, a small portion of the surveys were personally administered to farmers who were delivering grain to local grain elevators in southeastern Minnesota. Farmers were paid \$5 to complete the survey, which was comprised of seven questions pertaining to the producer's risk preferences and subjective beliefs about the relationship between fertilizer use and corn yields. A sample survey is provided in the Appendix. A total of 130 responses were obtained, with all respondents filling out the entire survey. Survey results were also compiled at the state level. The state level results did not differ from those of the entire population at common levels of statistical significance and are not reported.

²¹ Decision Commodities is a company based in Ames, IA. Decision Commodities offers market-based index contracts as risk management and marketing tools for farmers throughout Iowa, Illinois, Missouri, and North Dakota.

The survey design was modeled after that of Kahneman and Tversky's (1979) famous critique of expected utility theory. The first two questions of the survey were concerned with the farmer's preferences over monetary gambles with expected outcomes of equal value, and were included as "warm-up" questions for the farmer. The percentage of farmers from the total sample who chose each option is provided in parentheses following each choice. An asterisk denotes preference at a 5% significance level. Questions 1 and 2 asked the producer to choose between two monetary gambles.

Question 1:

- A. 50% chance of winning \$1000, 50% chance of winning nothing (20%)*
- B. \$450 with certainty (80%*)*

Question 2:

- A. 4% chance of winning \$12,000 and a 96% chance of losing \$500 (40%)*
- B. 95% chance of winning \$500 and a 5% chance of losing \$9,500 (60%*)*

The results of question 1 imply a strong preference for the certain amount versus a 50/50 gamble with a greater expected value (\$500 vs. \$450), implying risk aversion among the surveyed corn farmers. In question 2 there is a statistically significant preference for gamble B. Both gambles A and B in question two have expected values of zero. Gamble A provides a small chance of a large gain and a large chance of relatively small loss, whereas gamble B has a large chance of a small gain and small chance of a large loss. Noting that the variance of gamble A is greater than the variance of gamble B, farmer preferences are consistent with expected utility from a mean-variance standpoint.

Questions 3 through 5 asked the farmer to compare yield "gambles" rather than monetary gambles. The survey asked respondents to answer the questions assuming that they

did not have access to any type of government support programs, such as federal crop insurance. If the farmer answered the questions assuming he would be insured, low yield scenarios may not be valued at the full loss level.

Question 3 gives the farmer the options of a yield scenario with a large chance of yields slightly above the mean (180 bu/acre) and a small chance of very low yields (i.e. a crop failure), and a scenario with a small chance of a very large yield realization (i.e. a bumper crop) and relatively large chance of a yield realization slightly below the mean. Farmers had a statistically significant preference for the “small chance of a very large yield” gamble (B) over the “small chance of a very low yield” gamble (A). This would violate risk-aversion, in mean-variance terms, as the expected yield in both gambles is equal (180 bu/acre) with gamble B having a higher variance. The majority of surveyed farmers prefer yield risk above the mean relative to yield risk below the mean.

Question 3:

- A. 95% chance of 185 bu/acre and a 5% chance of 85 bu/acre (41%)*
- B. 4% chance of 300 bu/acre and a 96% chance of 175 bu/acre (59%*)*

Question 4:

- A. 95% chance of 185 bu/acre and a 5% chance of 85 bu/acre (18%)*
- B. 180 bu/acre (82%*)*

Question 5:

- A. 4% chance of 300 bu/acre and a 96% chance of 175 bu/acre (48%)*
- B. 180 bu/acre (52%)*

Questions 4 and 5 asked the farmers to compare the yield gambles from question 3 to certain yields at the mean level. In question 4 there is a significant preference for the certain

mean yield over the “small chance of a very low yield” gamble. However, the results for question 5 show no strong significant preference for the certain mean yield and the “small chance of a very large yield” gamble. Expected utility-maximizers would strictly prefer the certain mean yield over either gamble. While the farmers have a very strong preference for the certain mean yield over the “small chance of a very low yield” gamble, they are statistically indifferent between the certain mean yield and a “small chance of very high yields” gamble. This provides further evidence to the hypothesis that farmers do not consider above average yield realizations as risky.

Questions 3 through 5 also allowed for testing of the transitivity axiom of expected utility theory. Respondents who answered A-B-A or B-A-B for questions three through five violate transitivity. For example, denoting the utility of a gamble where the agent receives payout x (y) with probability π_1 (π_2) by $U(\pi_1, x; \pi_2, y)$, choosing option A for question 3 implies $U(0.95, 185; 0.05, 85) > U(0.04, 300; 0.96, 175)$. Similarly the choice of scenario B in question 4 implies that $U(1, 180) > U(0.95, 185; 0.05, 85)$ and the choice of scenario A in question 5 implies $U(0.04, 300; 0.96, 175) > U(1, 180)$. The latter two relations imply $U(0.04, 300; 0.96, 175) > U(0.95, 185; 0.05, 85)$, violating the respondent’s preference ranking implied by choosing scenario A in question 3²². The A-B-A type of transitivity violation was done by 14 (10.8%) of the respondents. Only one (<1%) farmer’s response qualified as a B-A-B type of transitivity violation.

Finally, the survey asked two questions regarding the farmer’s subjective beliefs of the effect of fertilizer application on yield risk and variability. While a slight majority of the

²² Indifference over all three gambles is also a possibility and would not imply a violation of transitivity.

surveyed farmers responded that they believe fertilizer increased yield variability, the significant majority of producers believe that fertilizer does not increase yield risk. The survey results imply that farmers do not equate yield “risk” with yield variability.

Question 6: Do you think applying more nitrogen fertilizer increases your yield risk?

Yes (28%)

No (72%)*

Question 7: Do you think applying more nitrogen fertilizer increases your yield variability?

Yes (56%)

No (44%)

Conclusions

The production literature is rich with studies examining the relationship between production uncertainty and optimal input use. Many authors have concluded that fertilizer is a risk-increasing input according to the original definition of Pope and Kramer (1979) using experimental yield response data (Ramaswami 1992; Just and Pope 1979). Empirical evidence tends to support these claims in that yield variability is generally found to be increasing in the amount of fertilizer applied (Roumasset et al. 1989). The Pope and Kramer definition is also applied in theoretical analyses where productive inputs are generally found to be risk increasing (Hurley et al. 2004; Hurley and Babcock 2003).

However, there are a number of empirical studies which illustrate that farmers consistently over-apply fertilizer (Yadav, Peterson, and Easter 1997; NRC 1993). The over-application of fertilizer is generally motivated as an act of self-protection (Ehrlich and Becker 1972) in response to uncertainty with respect to growing conditions (Below and Brandau 2001; Babcock 1992) and/or input availability (Babcock and Blackmer 1992, 1994),

or as the result of (incorrect) subjective beliefs regarding yield response (SriRamaratnam et al. 1989).

Obviously these views present somewhat of a paradox. This paper has shown that an input, nitrogen fertilizer, can be simultaneously defined as risk increasing and “over-applied” by both risk-neutral and risk-averse producers. All else equal, the risk-averse farmer will choose application rates below those of the risk-neutral farmer. This effect is attributed to the fact that the variance of yields is increasing in the level of fertilizer applied and the concavity of the utility function, the combination of which implies a positive marginal risk premium. However, if applied fertilizer is assumed to be an imperfect proxy for stochastic soil nutrients both risk-averse and risk-neutral farmers may apply fertilizer at rates exceeding the passive optimum, where the passive optimum is defined as the profit-maximizing application rate when soil nutrients are assumed to be a deterministic function of applied fertilizer. This response is due to the convexity of the (estimated) marginal product and (assumed) utility functions (Rothschild and Stiglitz 1971).

In either case, the optimal response of the agent (farmer) to uncertainty relies on the curvature of the production technology and preference relation, both of which can be easily manipulated by the researcher through estimation specification (technology) or assumption (technology and preferences). Furthermore, the use of various risk-increasing definitions by authors can lead to confusing and conflicting results in the literature. This paper has clearly shown the difference between the Pope and Kramer (1979) and Rothschild and Stiglitz (1971) approaches to defining the relationship between risk and input use. Pope and Kramer’s definition compares optimal input use across risk preferences while the Rothschild and Stiglitz approach is concerned with how the introduction of uncertainty may effect

optimal choices. The empirical results from this analysis show that fertilizer can be defined as risk-increasing by the Pope and Kramer definition and also be over-applied by both risk-averse and risk-neutral farmers due to the curvature properties of the production and utility relations first derived by Rothschild and Stiglitz (1971).

In addition to the empirical analysis, the results from a farmer survey were presented. The survey was designed to elicit information on farmer preferences over yield outcomes as well as their subjective beliefs about the relationship between risk and input use. The survey results imply that while farmers do prefer certain outcomes to gambles, they discount “risk” above the mean less than risk below the mean. Yield gambles with small chances of very large yield realizations are preferred to those with small chances of yields that would be defined as crop failures (where the expected yield is held constant). Moreover, while roughly 50% of the farmer respondents recognize that increased fertilizer use may increase yield variability, a much smaller percentage (28%) of respondents feel that additional fertilizer increases yield “risk.”

The survey results imply that the application of theoretical frameworks other than expected utility theory to decision making in agriculture are warranted. The application of behavioral methods in the general economics literature is rapidly growing. Similarly, there have been a significant number of recent studies applying experimental based behavioral methods to agricultural economics. Using these results as the basis for future models of choice under uncertainty in agriculture provides significant potential for further research and the re-examination of results obtained from previous theoretical frameworks (i.e. expected utility theory).

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Appendix

Sample Farmer Survey

The first two survey questions ask you to compare different situations involving monetary outcomes.

1. Which situation would you prefer?

___ A: 50% chance of winning \$1,000 and a 50% chance of winning nothing

___ B: Winning \$450 with certainty

2. Which situation would you prefer?

___ A: 4% chance of winning \$12,000 and a 96% chance of losing \$500

___ B: 95% chance of winning \$500 and a 5% chance of losing \$9,500

The following three survey questions ask you to compare different scenarios for the corn yields on your farm for a given year. Assume you do NOT have access to government support programs, such as Federal crop insurance, when answering these questions.

3. Which situation would you prefer for your average corn yields?

___ A: 95% chance of 185 bu/acre and a 5% chance of 85 bu/acre

___ B: 4% chance of 300 bu/acre and a 96% chance of 175 bu/acre

4. Which situation would you prefer for your average corn yields?

___ A: 95% chance of 185 bu/acre and a 5% chance of 85 bu/acre

___ B: 180 bu/acre

5. Which situation would you prefer for your average corn yields?

___ A: 4% chance of 300 bu/acre and a 96% chance of 175 bu/acre

___ B: 180 bu/acre

The final two questions of this survey ask you about the relationship between nitrogen fertilizer and your corn yields.

6. **Do you think applying more nitrogen fertilizer increases your yield risk?**

___ Yes

___ No

7. **Do you think applying more nitrogen fertilizer increases your yield variability?**

___ Yes

___ No

Note: The results of this ISU study are for research purposes only and your identity will remain confidential. The results of this survey will not be used for any type of commercial purpose such as the sales or promotion of any product or technology

CHAPTER 4.

A BAYESIAN AND SPATIAL APPROACH TO WEATHER DERIVATIVES: A FRAMEWORK FOR DEVELOPING REGIONS

A paper submitted to the *American Journal of Agricultural Economics*

Nicholas D. Paulson¹, Chad E. Hart, and Dermot J. Hayes

Abstract

There are a wide variety of farm and county level insurance programs available to livestock and crop producers in the United States and Canada. These programs rely on reliable long-term data for actuarial soundness. However, the expansion of crop insurance programs in other areas has been limited. Recently, the use of weather indexes as risk management tools has seen incredible growth. While the demand for weather based agricultural insurance in developing regions is limited, there exists significant potential for the use of weather indexes in developing regions (Varangis, Skees, and Barnett 2002). This paper proposes a Bayesian rainfall model which uses spatial kriging and Markov Chain Monte Carlo techniques to estimate unbiased rainfall histories from sparse historical data. The estimated history can then be used to develop actuarially sound weather based insurance. The method is validated using a rich data set of historical rainfall in Iowa. An example drought insurance policy is presented. The fair rates are calculated using Monte Carlo analysis and a historical analysis is carried out to assess potential policy performance. While the application is specific to forage production in Iowa, our method provides a framework which could easily be applied to other regions, such as developing areas, and for other crops.

¹ Lead researcher and primary author.

Introduction

Crop and livestock producers in the United States have access to a rich variety of yield and revenue insurance programs. Some of these products insure farm-level yields or revenues and others insure against declines in county-level yields and revenues. The actuarial success of these products depends on the availability of accurate yield histories at the farm and/or county levels and on efficient futures markets. Yield histories are needed to provide a yield guarantee, and futures prices are used to provide the price component of a revenue guarantee. These products are subsidized, and the federal government provides reinsurance. The provision of federal reinsurance is necessary in part because private sector reinsurers are wary of reinsuring the kind of systemic risks that can exist in agriculture (Miranda and Glauber 1997). An equally rich range of products is available in Canada, with provincial governments rather than federal governments providing institutional support.

Crop insurance (other than hail) has not developed at a similar rate outside of the United States and Canada. Possible reasons include a lack of government provision of reinsurance, a lack of accurate and long-term yield data at the farm or regional level, and a lack of interest among producers due to the availability of other revenue support programs. An alternative to traditional insurance based on farm or area yields is that of agricultural insurance based on weather events. Varangis, Skees, and Barnett (2002) note that while demand for weather risk management tools for agriculture in developed nations is limited because of the availability of subsidized insurance programs, there is considerable potential for their use in developing nations. This is attributable to many factors, including the lack of subsidized insurance programs and greater relative dependence on agriculture in developing areas, as well as the fact that weather-related disasters have a much larger adverse effect on

economies in developing regions. Moreover, since weather derivatives fall into the category of index products, the costs associated with administering their use are relatively low and adverse selection and moral hazard problems are virtually eliminated.

Our working hypothesis is that the provision of crop or revenue insurance programs would benefit agricultural producers in countries that do not already have access to these programs (i.e. developing areas). This is true because insurance programs typically increase the certainty equivalent returns (CER) by a multiple of the fair premium value (Hart, Babcock, and Hayes 2001)². It is also true because ongoing multilateral and bilateral trade agreements are having the effect of reducing other forms of income stabilization, and because the provision of income insurance programs can be viewed as “green box” support (non-trade-distorting) by the World Trade Organization (WTO 1994).

This article addresses two of the principal barriers to the international expansion of crop insurance programs previously described. The first barrier is the lack of high-quality, long-term data that can be used for insurance program development. We directly address this by developing a method to interpolate among available weather stations to “measure” actual rainfall at a particular site. We propose a Bayesian rainfall model that uses recently developed spatial kriging and Markov Chain Monte Carlo techniques. Using the proposed method, dense unbiased rainfall histories can be estimated from a sparse grid of historical data. The rainfall distributions that we generate can then be used to find the actuarially fair rates for a weather derivative that is designed to indemnify producers against drought.

². Note that this result was shown for the specific case of insurance for livestock and was not generalized to all insurance programs.

The second problem is finding a private sector substitute for the reinsurance that is currently provided by the government in the United States and Canada. While we do not explicitly outline a reinsurance scheme, the method we propose generates rainfall distributions for non-sample sites with the spatial correlation structure between observed locations. The availability of this spatial correlation structure is key because it provides the information needed by a reinsurer to separate systemic and non-systemic risks.

In order to validate the proposed method, we apply it to a problem in which the answer is already known. Iowa has a rich series of rainfall data from numerous weather stations, and this allows us to first predict the actual rainfall at each station and then compare the predicted and actual values. We show that the method is accurate and unbiased and that we can successfully uncover the spatial correlation structure across sites. We also show that it is possible to develop and rate a practical drought insurance policy using the Iowa rainfall data. While the application is specific to forage production in Iowa, the methods used can easily be generalized to rate a weather derivative for a variety of crops and regions, including developing areas. More importantly, spatial kriging can generate unbiased rainfall histories in areas for which the density of historical data may be quite low. This would allow weather risk products to be accurately priced for many developing areas for which historical information may be scarce.

Literature Review

Demand for Weather Derivatives in Agriculture

The largest obstacles facing development of weather derivative products in developing areas are basis risk and the lack of historical weather data. Basis risk, in the specific case of

rainfall and its effect on agriculture, refers to the relationship between the precipitation measured at the weather station and the production or revenue on the farm. Basis risk is more problematic for individual purchasers whose risk exposure is more centralized (Varangis, Skees, and Barnett 2002). The users, or purchasers, of weather derivatives would like to minimize the basis risk involved with the use of weather data collected at a site that does not necessarily correspond with their exposure location (Dischel 2000). Martin, Barnett, and Coble (2001) propose that weather derivative basis risk may be reduced considerably through a portfolio holding of various weather derivatives based on several surrounding weather stations. Dischel (2002) notes that

“Farmers, growers and hydroelectric generators would like to have contracts written on rain falling on their fields, in their groves or over their watersheds. This is generally impossible because the market needs long and accurate measurement records to assess the value of a weather derivative, and unaffiliated parties do not generally compile measurement records at these locations.”

Other studies have explored the potential demand for agricultural insurance based on precipitation. Sakurai and Reardon (1997) and Gautam, Hazell, and Alderman (1994) use household survey data to estimate latent demand for drought insurance in West Africa and southern India, respectively. Using a set of reduced-form equations resulting from the optimality conditions of a dynamic household optimization problem, both studies estimate a positive latent demand for drought insurance. Additionally, it is estimated that the insurance would be implementable on a full-cost basis. McCarthy (2003) estimates the demand for rainfall-based insurance contracts for four regions in Morocco, finding that the median willingness to pay for rainfall-based insurance was 12%–20% above the fair value of the contracts.

Weather Based Insurance

Weather patterns tend to exhibit positive spatial correlation, making losses more volatile from the perspective of the insurer, increasing the cost of maintaining adequate reserves to cover potential losses from systemic events. Thus, insurance may not be the optimal mechanism for providing efficient risk sharing (Skees and Barnett 1999). However, if the insurer can cover an area large enough to diversify the systemic risk of weather events or has access to an adequate reinsurance program, an insurance mechanism should be feasible and implementable (Duncan and Myers 2000).

Despite the largely systemic component of weather risk, there have been many recent studies examining the feasibility of developing agricultural insurance based on weather indexes. Martin, Barnett, and Coble (2001) outline various option structures for precipitation insurance and provide a rating method application for cotton in Mississippi. Skees et al. (2001) investigate the development of drought insurance based on a rainfall index in Morocco and find that the product would be both feasible and of significant benefit to Moroccan farmers. Turvey (1999, 2001) also discusses the application of weather derivatives in agriculture by rating various examples of rainfall and temperature options for various locations in Canada. To relate crop yields to weather events, Turvey (2001) examines the correlation of corn, soybean, and hay yields with measures of both rainfall and temperature. Temperature was found to be highly correlated with corn and soybean yields, while precipitation showed more correlation with hay yields.

Precipitation insurance policies have also been explored and utilized in other countries. Argentina, Ethiopia, Mexico, Morocco, Nicaragua, and Tunisia have all tested the feasibility of weather-based insurance products for agriculture (Varangis 2001), while

Australia is currently exploring the possibility of developing rainfall insurance (Plate 2004). Two Canadian provinces, Ontario and Saskatchewan, have precipitation insurance products on the market. The use of precipitation-based insurance in the Canadian provinces is attributed to the high correlation between cattle pasture productivity and rainfall (Varangis 2001).

Rainfall Interpolation

There is an extensive literature focused on rainfall interpolation techniques. The simplest method sets the value of rainfall at out-of-sample locations equal to the rainfall recorded at the nearest observed site (Thiessen 1911). In 1972 the National Weather Service adopted another method, with rainfall estimated as a weighted average of surrounding observed values, in which the weights were inversely proportional to the squared distances from the unobserved site (Bedient and Huber 1992). This method is not useful for our purposes because each site is treated as an independent observation and provides no information on the spatial correlation structure of rainfall. More recently, advances in the area of geostatistics have created more statistically sophisticated interpolation methods through the use of kriging. Kriging, or optimal prediction, refers to the practice of making inferences on unobserved values of a random process given data generated from the same process (Cressie 1993). In practice, kriging techniques form a predictor that is equal to a weighted average of the data in the sample. The weights used in the average are determined from the correlation structure of the process, which may be given, assumed, or estimated from the data. Kriging techniques have been shown to provide predictors that are both statistically unbiased and efficient.

While kriging methods provide statistically attractive properties, they can also require a significant amount of computing time and effort. Thus, many studies have focused on the comparison of point estimates obtained from kriging to the estimates based on simpler interpolation approaches. While many authors have shown that kriging techniques provide better estimates than do simpler methods (Tabios and Salas 1985), others have found that the results depend critically on the density of the sampled locations. In general, studies have shown that kriging dominates the simpler interpolation methods for areas with smaller sampling densities while the methods are fairly equivalent for areas with sampling grids of higher density.

Cressie discusses various types of kriging, which differ with respect to the underlying assumptions for the stochastic process. In general, the spatial process is modeled as the sum of a mean and a spatially correlated error component. Bayesian kriging assumes that the mean and error components are random and independent while recognizing that the model parameters are themselves stochastic. Given appropriate priors for the parameters of the mean and error structure components, the optimal Bayesian predictor for out-of-sample locations can be found and has been shown to be superior to other kriging methods (Cressie 1993).

Markov Chain Monte Carlo Methods

While point estimates for the conditional means and variances of a stochastic spatial process in a Bayesian model can be derived explicitly given appropriate distributional assumptions (see, e.g., Kitanidis 1986), an alternative approach is to sample directly from the posterior distribution using Markov Chain Monte Carlo (MCMC) techniques. MCMC methods are often employed when explicit evaluation of complex and high dimensional

integrals is not possible. Under these circumstances, MCMC techniques provide an alternative to more traditional numerical or analytic methods of integration. MCMC methods differ from traditional Markov chain theory in that the process's stationary distribution is used to identify the transition distribution rather than the reverse problem (Brooks 1998).

The theorem on which MCMC methods are based states that any chain that is both irreducible and aperiodic will have a unique stationary distribution to which the t -step transition kernel will converge as t approaches infinity (Brooks 1998). In practice, chains are generated either using a single transition kernel or, in many cases, using a combination of multiple sampling algorithms. The two most common transition kernels are the Gibbs sampler and the Metropolis-Hastings algorithm. The Gibbs sampler operates by splitting the current state vector into a number of components while updating each component separately in turn. The Metropolis-Hastings algorithm differs from the Gibbs sampler in that it is a generalized rejection sampler in which drawn values are corrected to match asymptotic properties of the stationary distribution.

Implementation is achieved by specifying starting values and non-informative priors for each process variable. There are several implementation issues involved with MCMC techniques. The chains are naturally autocorrelated because of the sampling algorithms' dependence on the previous step. This is generally addressed by "thinning" the chains to save only a portion of the realizations. Actual convergence is also an issue and is usually addressed by running multiple chains from different starting values and comparing running plots of the realizations to ensure convergence to the same distribution. These plots are also used to determine the number of initial values to discard, referred to as "burn-in" values, so that the portion of the chains used for analysis are an accurate representation of the stationary

distribution. Sufficient chain length is also under debate. In general, the minimum chain length depends on the problem at hand and is increasing in the standard deviation of the sample mean of some function of the iterations and decreasing in the level of autocorrelation between consecutive realizations. Once convergence and stationarity have been determined, point estimates can be computed as sample moments from the sampling distributions. See Brooks (1998) and Gilks, Richardson, and Spiegelhalter (1996) for more detailed descriptions of the theory behind MCMC methods and implications for empirical implementation.

The Rainfall Model

Following Cressie, and Kitanidis (1986), in order to derive an empirical Bayes predictor for rainfall, let y_i denote observed rainfall at location i and assume that the actual rainfall at a given site is determined by the sum of a mean or drift process, μ , and a spatially correlated error process, ε , which are both functions of site-specific measures X and K model parameters θ .

$$(1.1) \quad y_i = \mu(X_i, \theta) + \varepsilon(X_i, \theta).$$

Applying Bayes' theorem, the posterior distribution for the stochastic model parameters θ conditional on observed rainfall y is the product of the likelihood function and the prior distribution normalized by an appropriate constant.

$$(1.2) \quad p(\theta | y) = \frac{p(y | \theta)p(\theta)}{\int_{\theta_1} \dots \int_{\theta_k} p(y | \theta)p(\theta)d\theta_k \dots d\theta_1}$$

For any unobserved location j , the distribution of rainfall \tilde{y}_j conditional on observed rainfall at a sample of N locations, $y = y_1 \dots y_N$, is given by

$$\begin{aligned}
(1.3) \quad p(\tilde{y}_j | y) &= \int_{\theta_1} \dots \int_{\theta_k} p(\tilde{y}_j, \theta | y) d\theta_k \dots d\theta_1 \\
&= \int_{\theta_1} \dots \int_{\theta_k} p(\tilde{y}_j | \theta, y) p(\theta | y) d\theta_k \dots d\theta_1 \\
&= E_{p(\theta|y)} [p(\tilde{y}_j | \theta, y)].
\end{aligned}$$

Thus, the posterior distribution for any \tilde{y}_j given y is taken as the expected value of the posterior conditioned on y and θ with respect to the posterior distribution of θ conditioned on y . If the only variable of interest is rainfall at an unobserved site \tilde{y}_j , the posterior distribution given in equation (1.3) is all that is required. The information contained in the posterior defined by equation (1.3) could be used to accurately price a weather derivative for any unobserved site j . For example, the mean rainfall $E[\tilde{y}_j]$, which may be of interest as an option strike or insurance guarantee, would be given by

$$(1.4) \quad E[\tilde{y}_j] = \int_{\tilde{y}_j} \tilde{y}_j p(\tilde{y}_j | y) d\tilde{y}_j .$$

However, for reinsurance purposes it is critical to have information on how rainfall is jointly distributed across space. The intratemporal spatial correlation structure captured by the joint posterior distribution of unobserved sites provides the information needed by a reinsurer who owns a portfolio of policies rated for individual locations. The joint posterior for a collection of J unobserved locations, $\tilde{y} = \tilde{y}_1 \dots \tilde{y}_J$, is given by

$$\begin{aligned}
(1.5) \quad p(\tilde{y} | y) &= \int_{\theta_1} \dots \int_{\theta_k} p(\tilde{y}, \theta | y) d\theta_k \dots d\theta_1 \\
&= \int_{\theta_1} \dots \int_{\theta_k} p(\tilde{y} | \theta, y) p(\theta | y) d\theta_k \dots d\theta_1 \\
&= E_{p(\theta|y)} [p(\tilde{y} | \theta, y)].
\end{aligned}$$

The marginal distribution for any unobserved site j is then given by

$$\begin{aligned}
(1.6) \quad p(\tilde{y}_j | y) &= \int_{\tilde{y}_1} \dots \int_{\tilde{y}_{j-1}} \int_{\tilde{y}_{j+1}} \dots \int_{\tilde{y}_j} \int_{\theta_1} \dots \int_{\theta_K} p(\tilde{y}, \theta | y) d\theta_K \dots d\theta_1 d\tilde{y}_j \dots d\tilde{y}_{j+1} d\tilde{y}_{j-1} \dots d\tilde{y}_1 \\
&= \int_{\tilde{y}_1} \dots \int_{\tilde{y}_{j-1}} \int_{\tilde{y}_{j+1}} \dots \int_{\tilde{y}_j} \int_{\theta_1} \dots \int_{\theta_K} p(\tilde{y} | \theta, y) p(\theta | y) d\theta_K \dots d\theta_1 d\tilde{y}_j \dots d\tilde{y}_{j+1} d\tilde{y}_{j-1} \dots d\tilde{y}_1.
\end{aligned}$$

The expressions given in equations (1.5) and (1.6) are Bayesian distributions of rainfall at unobserved locations given the rainfall data from the observed sites, y . These distributions account for parameter uncertainty and differ from a non-Bayesian approach in which point estimates for the parameters might be treated as known (Kitanidis 1986). The information on the spatial correlation of rainfall across space is given in both the joint posterior distribution of rainfall at the unobserved sites outlined in equation (1.5) and the posterior distribution for the model parameters given in equation (1.2).

Given the potential size of the integral in equation (1.6), many cases may arise in which explicit evaluation would be impossible. As an alternative, MCMC methods can be used to simultaneously generate Markov Chains of both the model parameters from the posterior distribution in equation (1.2) and rainfall estimates for any number of unobserved locations from the posteriors given in equations (1.5) or (1.6).

Implementation Example

To estimate the model, the structure of the mean and error processes must be specified. Appropriate starting values and priors for the model parameters and rainfall at the unobserved sites are also needed. To provide an example of how the rainfall model could be implemented, suppose that the mean component of rainfall at any given site is a linear

function of its geographic coordinates. Furthermore, assume that the spatial correlation structure for a group of locations can be summarized by an exponential correlogram³.

$$(1.7) \quad \mu(X_i, \theta) = \theta_0 + \theta_{lat} X_i^{lat} + \theta_{long} X_i^{long}$$

$$(1.8) \quad \Sigma_{ij} = f(d_{ij}, \varphi, \kappa) = \exp\left(-(\varphi d_{ij})^\kappa\right).$$

Under the exponential specification, the correlation between rainfall at locations i and j , Σ_{ij} , is a function of the Euclidean distance d_{ij} between the two locations. The exponential correlogram assumes that the correlation between observations declines with the distance between the observations. This property makes it a natural choice for modeling the spatial correlation structure of weather events across a region. The parameters κ and φ are measures of spatial smoothing and decay, respectively. The smoothing parameter, κ , is bounded between zero and two, with larger values indicating higher levels of spatial smoothing. A value of κ equal to two implies the Gaussian correlation function. The decay parameter, φ , is bounded below at zero and indicates the degree of decline in correlation between two locations with distance. A larger (smaller) value of φ indicates a faster (slower) decline in correlation as distance increases (Thomas et al. 2004). Thus, larger estimates for φ indicate a smaller degree of similarity between nearby stations.

Given observed rainfall for a set of locations and their geographic coordinates, ordinary least squares estimates of the mean process parameters would provide appropriate starting values. Starting values for the correlogram parameters can be obtained using maximum likelihood estimates if multiple periods of data are available. Alternatively, multiple chains can be generated from a wide range of starting values to ensure model

³. To clarify notation, the parameterization given in equations (1.7) and (1.8) imply that the model parameters are given by $\theta = \{\theta_0, \theta_{lat}, \theta_{long}, \varphi, \kappa\}$.

convergence of the model to the same stationary distribution regardless of starting values. Prior selection should be limited to diffuse distributions, such as the uniform, to prevent any effect of prior specification on the results of the model. We now provide an application of our proposed method using rainfall data from Iowa.

Data

State-level monthly precipitation totals for Iowa were obtained from the National Oceanic and Atmospheric Administration's National Climatic Data Center (NCDC). The historical series of precipitation totals for all sequential combinations of months were compared to historical per-acre hay yields as reported by the National Agricultural Statistics Service (NASS) for the state of Iowa. The April through December time period showed the highest correlation between cumulative precipitation and hay yields for Iowa and was adopted as the coverage period for the weather derivative example. In addition to aggregated state-level data, the NCDC reports data from thousands of individual weather stations located throughout the country. The full data set of Iowa weather stations was condensed to exclude those weather stations that did not have complete precipitation records for the months included in the coverage period (April–December) for the entire thirty-year period from 1973 to 2002. At the time of data collection, the last monthly recording was for August 2003, hence the use of 1973-2002 data to calculate the thirty-year average precipitation levels guaranteed by the policy. Given the data requirements, the number of usable weather stations was reduced to sixty-seven in the state of Iowa. The grid of sixty-seven weather stations provides a relatively dense sampling grid in comparison to previous studies (Tabios

and Salas 1985; Dirks et al. 1998). The distance between adjacent weather stations averages 20 miles, with a maximum (minimum) distance between weather stations of 50 (7) miles.

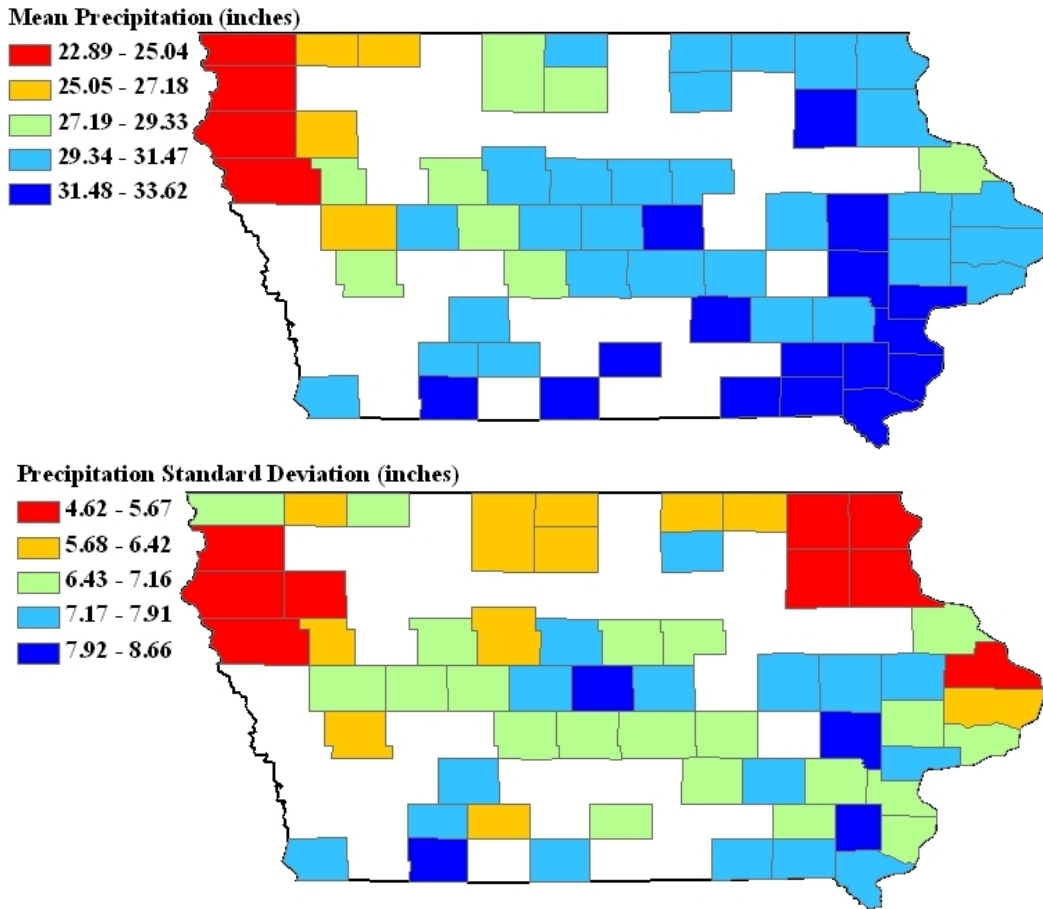


Figure 1. Average and standard deviation of reported precipitation at Iowa weather stations, 1973-2002

Figure 1 shows the means and standard deviations of reported precipitation levels, in inches, for the counties in which the weather stations are located. The weather station data show that the northwest section of Iowa tends to be the driest, with more precipitation, on average, being reported as one moves toward the southeast section of the state. Precipitation variability, as measured by the standard deviation of reported precipitation, follows a similar

pattern across the state, with lower variability in the northern section of the state and higher variability in the central and southern regions.

Two additional issues arose with the weather station data. First, for some stations and months, only estimated precipitation values were available. These estimated values were assumed to be unbiased and were left unchanged. Second, for some other stations and months, the precipitation values were reported as incomplete. For these incomplete months, the NCDC indicated that somewhere between one and nine days of information were missing from the reported precipitation value. In order to conserve these data points, it was assumed that the incomplete months were missing the average of five days of precipitation information and that the precipitation amount during those five days was equal to the five-day average precipitation amount for the month based on the reported total.

The coordinates of the geographical centers of each county in Iowa, measured in degrees of latitude and longitude, were calculated from a data file created by Giglierano and Madhukar (1990). This yielded ninety-nine county reference points, or sample “farms,” where rainfall could be interpolated to rate the weather derivative. The geographic coordinates of each of the sixty-seven weather stations in Iowa were obtained from the NCDC. The Euclidean distances between the weather stations and reference points were calculated using the coordinate data.

Policy Structure

The rainfall guaranteed under the policy was taken as the thirty-year average of recorded precipitation for the area over the insurance period, which is patterned after the thirty-year climate normals used by the NCDC. The indemnity (I) takes the form of an

exotic put option on the 30-year average rainfall guarantee. The indemnity structure is similar to an example outlined by Martin, Barnett, and Coble (2001).

$$(1.9) \quad I = \text{Max} \left[0, \text{Min} \left(\left(L * F * \left(C - \frac{R_A}{R_{30}} \right) \right), L \right) \right]$$

where L = liability value
 F = indemnity factor
 C = coverage level ($C \in [0,1]$)
 R_A = actual rainfall
 R_{30} = the rainfall guarantee

Indemnities are triggered when actual precipitation is less than a selected percentage (the coverage level, C) of the historical average precipitation. The percentage shortfall in precipitation is translated into a shortfall in liability value, with the indemnity paid equal to the liability shortfall.

The indemnity factor F was created to translate precipitation shortfalls into liability losses. A regression relating NCDC precipitation levels to NASS hay yields was estimated for Iowa. To put all variables on a percentage basis, ratios were created for each variable. The precipitation ratio (RR) is the ratio of the current year's precipitation to the thirty-year average. The hay yield ratio (YR) is the ratio of the current year's reported hay yield to the ten-year average hay yield. It is assumed that excessive amounts of rainfall can also cause crop losses. Since the intent of the application was to provide coverage against drought risk, only years in which rainfall was below the thirty-year average were used in estimating the regression relationship. Table 1 reports the regression estimates.

$$(1.10) \quad (YR_i | RR_i \leq 1) = \alpha + \beta * (RR_i | RR_i \leq 1) + \varepsilon_i$$

Table 1. Hay Yield – Precipitation Regression Coefficient Estimates (Standard Errors)

$\hat{\alpha}$	$\hat{\beta}$	R ²
-0.34 (0.20)	1.52 (0.22)	0.85

The sign of the estimated slope coefficient was as expected, with precipitation shortfalls leading to a reduction in hay yields below the average level. The results exhibit fairly strong yield movements in Iowa, with a 1% drop in precipitation from the thirty-year average resulting in a 1.52% drop in hay yields below the ten-year average hay yield. The indemnity factor F was taken as the slope coefficient estimate. Thus, the policy pays 1.52% of the liability for every 1% drop in precipitation below the guaranteed historical average.

Results

Kriging

For each year in the data a sample from the posterior distributions of each model parameter and rainfall for each of the ninety-nine sample farms were generated. The program was set to estimate rainfall for each sample site individually, significantly reducing the order of integration.⁴ The latitude and longitude coordinates for each of the weather stations and reference points were normalized to make the southwest corner of Iowa the grid origin. Sample autocorrelation plots from initial sample iterations exhibited autocorrelation through ten lags in the chains. To obtain a closer approximation to an independent sample,

⁴. Although this approach saved considerable time, the correlation structure of the Markov chains for unobserved rainfall across space for any given year was lost. While this information was not critical to the specific application of policy rating, the spatial structure of the rainfall distributions would be of considerable interest for reinsurance purposes. Information on the correlation structure of rainfall across space is still provided by the correlogram parameter estimates.

the chains were thinned to save every tenth iteration. To assess convergence, three chains of 55,000 iterations were run from different starting values. The first 5,000 iterations of each chain were discarded to minimize the impact of the starting values. As a diagnostic for sufficient chain length, we confirmed that the Monte Carlo error for the samples was less than 5% of the sample standard deviation.⁵ The estimation process yielded Markov chains of 5,000 rainfall and parameter samples for each year in the data. The point estimates for the model parameters and unobserved rainfall at the reference points were taken as the sample means from the Markov chains. The estimated thirty-year means and standard deviations of precipitation are illustrated in figure 2, and are very similar to those in the actual weather station data illustrated in figure 1. Furthermore, cross-validation confirmed that the kriging results were statistically unbiased estimates of actual rainfall, while the average standard deviation of the bias estimates⁶ was 3.01 inches of rainfall. These results can be interpreted as upper bounds on the performance of the model, as the cross-validation results come from a sampling grid of lower density.

Summary statistics of the parameter-point estimates for the mean process and the correlogram are given in table 2. The point estimate for θ_0 can be interpreted as a rainfall estimate for the southwest corner of the Iowa grid and averaged just under 29 inches of rainfall, which is consistent with the true thirty-year means from weather stations in that region. The point estimates for θ_{lat} and θ_{long} indicate that, on average, precipitation declines by 1.46 inches for every degree of latitude as you move north and increases by 0.85 inches

⁵. The Monte Carlo error is a measure of the deviation of the sampled mean from the mean of the true posterior distribution. See Gilks, Richardson, and Spiegelhalter 1996 and Brooks 1998 for further discussion on diagnostics in MCMC applications.

⁶. Cross-validation refers to estimating the model sequentially for each observed site, or weather station, based on the data for the $N-1$ remaining stations. The bias estimates refer to the difference between the rainfall estimates from the model and the actual rainfall recorded at that weather station for the given year.

for every degree of longitude as you move east. These results are also consistent with the relationship between average rainfall amounts and location in the state of Iowa, as depicted in figure 1.

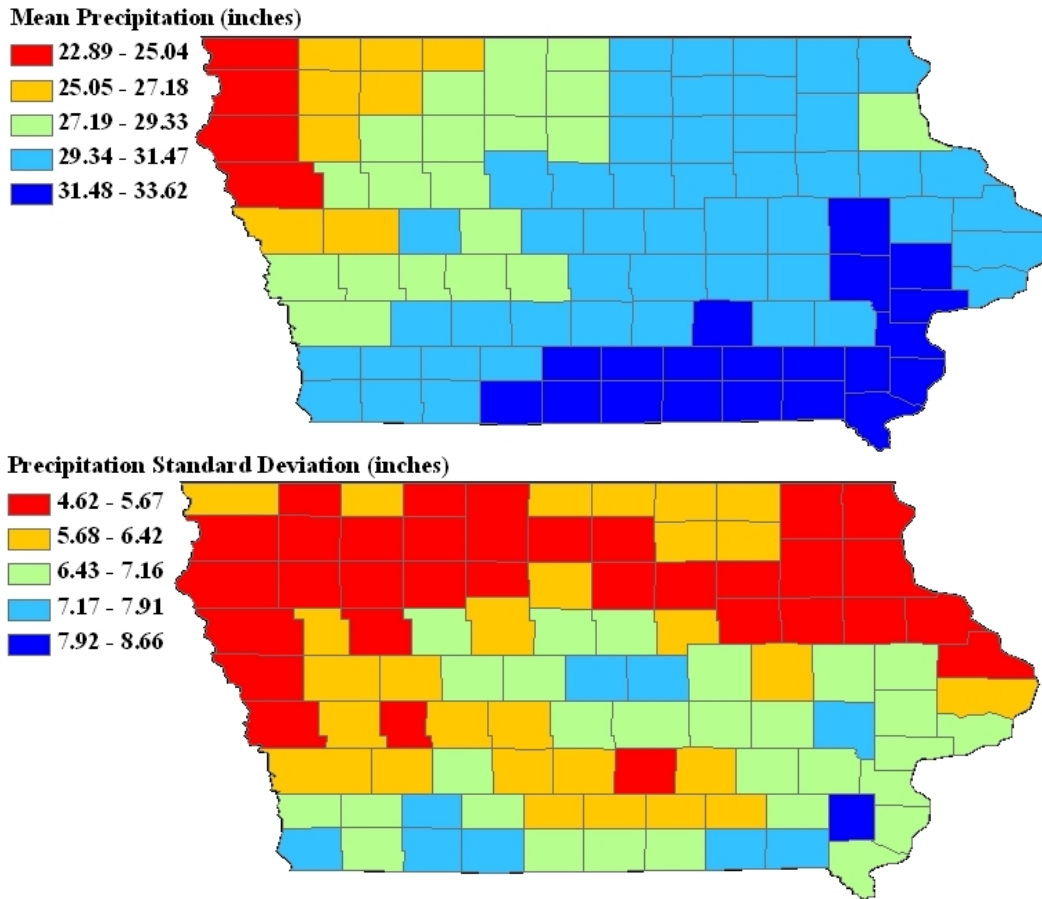


Figure 2. Average and standard deviation of estimated precipitation for Iowa counties, 1973-2002

The point estimate of the smoothing parameter, κ , ranged from 0.56 to 1.66, with an average value of 1.01. Point estimates for the decay parameter, φ , varied within a considerable range from 0.48 to 11.34, with an average value of 3.58. A larger point estimate for φ indicates a weaker spatial correlation structure in the rainfall data for the given

year, implying the occurrence of localized storms and volatile rainfall amounts across the grid over the corresponding time period.

Table 2. Summary Statistics of Rainfall Model Parameter Point Estimates

	θ_0	θ_{lat}	θ_{long}	κ	φ
Mean	28.91	-1.46	0.85	1.01	3.58
Median	28.07	-1.89	0.59	0.97	2.03
Standard Deviation	6.89	1.90	0.96	0.24	3.37
Minimum	18.20	-4.70	-0.56	0.56	0.48
Maximum	43.16	2.74	2.85	1.66	11.34

Inverse Distance Weighting

For comparison purposes, a simple inverse distance weighting (IDW) scheme was also used to interpolate precipitation in the Iowa counties using the historical data. For each county reference point, the weights assigned to the weather stations were equal to the inverse of the Euclidean distance between the reference point and the weather station normalized by the sum of weights for all weather stations. Using cross-validation, the number of surrounding weather stations to use in the interpolation was varied from the nearest station to the entire set of surrounding weather stations. All IDW estimators were found to be statistically unbiased with gains in efficiency up to four surrounding stations. Using the four nearest stations, rainfall at each of the county reference points was interpolated over the 30 years of data. The results were nearly identical to those from the kriging model with the 30-year averages differing by less than 0.7 inches for all county reference points. Coefficients of variation were also nearly identical for the two methods, implying that the insurance rates generated from either method would also be nearly identical.

The “equivalence” of the two methods should be interpreted with care. The IDW results only provide point estimates for unobserved rainfall and are not guaranteed to be unbiased or efficient estimators. Moreover, the kriging model provides a richer set of results including empirical distributions of rainfall at each of the out-of-sample locations and parameter estimates which define the correlation structure of rainfall across space. This information has significant value for estimating the magnitude of basis risk as well as implications for accurately rating a reinsurance program.

Insurance Rates

To rate the insurance policy, Monte Carlo analysis was used, assuming that rainfall over the coverage period follows the gamma distribution defined by the historical rainfall parameters. Using a method-of-moments approach, gamma distributions were fit to the historical rainfall means and standard deviations implied by the kriging estimates for the ninety-nine reference points. For each method, 5,000 random draws were taken from each of the specified gamma distributions. The policy was then rated by taking the average indemnity value over the 5,000 rainfall draws for each of the ninety-nine reference points. Note that this risk-neutral pricing approach does not incorporate a market price of risk into the fair premium rates.

The choice of gamma distributions was based on the prevalence of this distributional choice for precipitation in the scientific and agricultural literature (Barger and Thom 1949; Ison, Feyerherm, and Bark 1971; Martin, Barnett, and Coble 2001). Using the method proposed by Moschini (1990), nonparametric kernel densities were fit to each of the 30 year precipitation histories for the weather stations and compared with the gamma distributions implied by the sample moments. The gamma density plots were very similar to the

nonparametric estimates and were determined to provide an excellent fit to the data. As an example, the gamma distribution and the nonparametric density for the Chariton weather station are illustrated in figure 3.

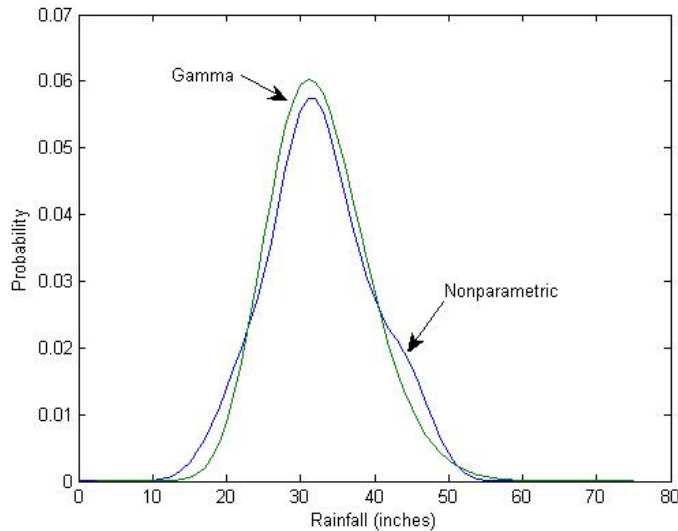


Figure 3. Gamma and Nonparametric Rainfall Densities, Chariton Weather Station

Given the rainfall insurance structure, the MCMC simulation results, and the gamma distributional assumption, Iowa premium rates average 12.4% under full coverage. The average premium rate across the Iowa reference points is equal to 1.2% for 75% coverage. At 75% coverage, the highest rate is 2.35% in Southeast Iowa at the Taylor county reference point, while the lowest premium is 0.3% in Northeast Iowa at the Clayton county reference point. These results are expected, as the lowest implied precipitation coefficient of variation (15.8%) is at the Clayton County reference point, while the largest implied coefficient of variation (24.6%) is at the Taylor County reference point. Figure 4 maps the premium rates across Iowa at a 75% coverage level. In general, premium levels are the lowest in the Northeast section of the state, with areas of relatively larger premium levels located in various locations throughout the rest of Iowa.

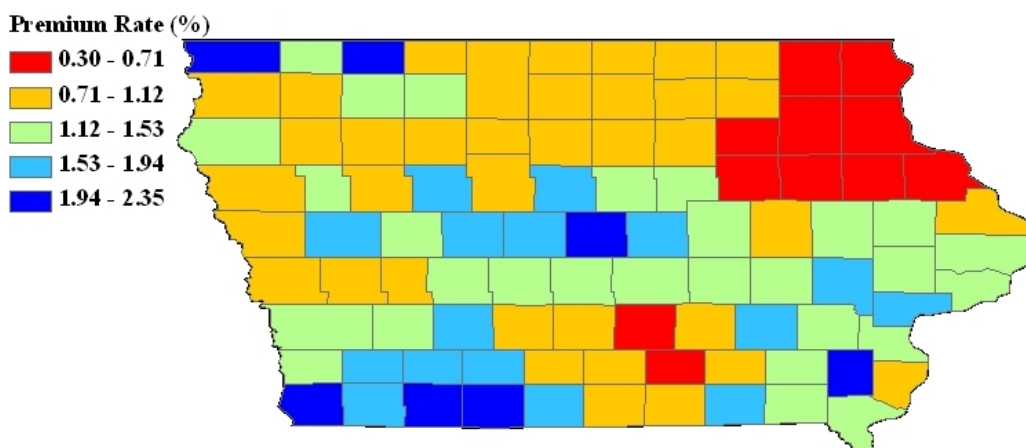


Figure 4. Premium rates at 75% coverage

Historical Analysis

A historical analysis of the insurance policy was constructed for the thirty years of available data. Precipitation estimates were taken from the kriging results and used to calculate the indemnity level for each year. The top panel of figure 5 maps the policy's loss-cost, the ratio of indemnity to liability, at 75% coverage for 2000. Precipitation was below 75% of the thirty-year average for a pocket of counties in southwestern Iowa, triggering indemnity payments. Up to 26% of the total liability covered under the policy would have been paid out in indemnities in 2000. Loss ratios, the ratio of indemnities to premiums, would have been as high as 18 in some counties, with an average loss ratio of one across the state. Thus, in 2000, the variability of rainfall was diversifiable across the entire state.

The bottom panel of figure 5 maps loss-cost values for each of the counties for 1988, a drought year throughout the Midwest. Indemnity payments would have been triggered in all but the Northwest quadrant of the state, with loss-cost exceeding 40% of the liability insured in some areas. Loss ratios in 1988 would have exceeded 50 in select regions with an average of 15.75 across the entire state. In general, the policy tends to pay indemnities in

concentrated areas and at fairly high loss ratios. At higher coverage levels the loss regions expand to cover larger areas across the state. These results are expected given the spatial nature of weather events.

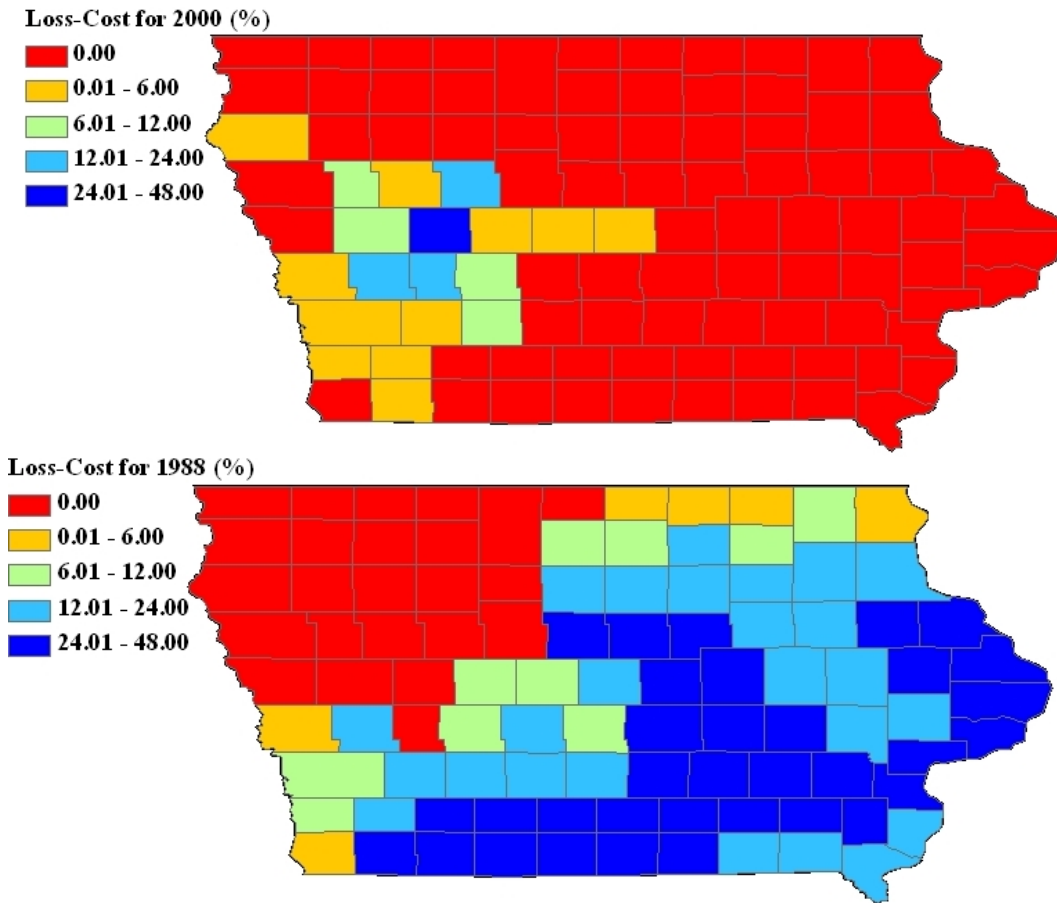


Figure 5. Loss-costs in Iowa counties for 2000 and 1988

While the policy is theoretically rated to yield a loss ratio of one over time for any given location, the systemic nature of weather risk requires a large geographic area of coverage to provide proper risk pooling and insurability for any given year. These results suggest that any party offering this type of coverage should either hold a diversified portfolio of policies written across a spatially diverse area or hold sufficient reserves (or reinsurance) to cover years when rainfall is well below the level of the guarantee.

Conclusions

The institutional models used to provide farm and county-specific crop insurance in the United States and Canada depend on access to accurate yield data and the provision of public sector reinsurance. These models are not directly applicable outside of the United States and Canada, because accurate yield histories are generally not available and because public provision of reinsurance is not affordable. These barriers are most acute in less-developed countries that depend heavily on agriculture. As a result there are very few examples of farm or county-level yield insurance products outside of the United States and Canada.

Weather derivatives have many properties that make them suitable as a basis for the expansion of crop insurance to areas where it is currently not available. Weather derivatives are easy to adjust and they greatly reduce the potential for moral hazard and adverse selection. A major problem with weather derivatives is that they cannot be used to exactly replicate farm-level yield performance. In this article we show how to use existing weather station data to minimize this weather basis risk.

A second difficulty faced by those who wish to introduce crop insurance in other countries is that public reinsurance is often not available. Private reinsurers and international agencies might be willing to provide reinsurance as long as they can be convinced that they can spread the risk across space; however, this requires knowledge of the spatial correlation of losses. The method we propose also provides this information.

In order to illustrate our method at work, we have implemented it for drought in Iowa. We show that we can simulate actual rainfall at a particular point in an efficient and unbiased fashion. We use the data to calculate fair rates for a drought insurance product and we show

that the cross-location correlation structure can be estimated for various uses, including the possible development of a reinsurance program.

Given the systemic nature of weather, loss areas over the period analyzed tended to be geographically concentrated, exhibiting high loss ratios. While weather insurance policies can be fairly rated for any given location over time, a sufficiently large geographic coverage area would generally be required for sufficient risk pooling within a given contract year. Thus, a drought insurance policy such as this may be more suited for inclusion in the book of business of a large reinsurer or an international agency. The rainfall interpolation methods utilized in this study could be widely applied to data in other areas to develop other types of derivatives, including insurance for various agricultural and non-agricultural applications.

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CHAPTER 5. GENERAL CONCLUSIONS

Conclusions

Agriculture is an industry plagued with numerous sources of risk and uncertainty. Both producers and buyers of agricultural commodities are forced to make decisions in a complex environment of volatile prices, uncertain growing conditions, and ever changing consumer preferences toward agricultural goods. In fact, it is difficult to imagine another industry where risk and uncertainty would play a larger role than in agriculture. This dissertation has analyzed three different areas related to risk and uncertainty in agriculture.

The focus of Chapter 2 was on contract structures in specialty grains markets and their implications on the sharing of production risk between the buyer and sellers. Using a theoretical model, acreage and bushel contract structures were compared in an environment characterized by a risk-neutral, monopsonistic processor and risk-neutral producers. The conventional wisdom in the contracting literature is that acreage contracts are preferred because producers are assumed to be relatively more risk-averse than processors.

However, the market environment characterized by the assumptions of the model in Chapter 2 led to the opposite result. The bushel contract structure was shown to be preferred by the processor. Moreover, it was shown that the bushel contract structure may Pareto dominate the acreage contract as producer profits may also be greater under bushel contracts.

When using an acreage contract, the processor is obligated to purchase every bushel of the specialty crop produced by contracted farmers. By using a bushel contract, the processor is able to shift a portion of the production risk to the producers. In years of low yields the processor receives an underage penalty payment from producers, while in years of

high yields the processor benefits from the *ex post* spot market by being able to purchase additional bushels of the specialty crop (up to his plant capacity) at the producer's salvage value (the commodity price). The producers are compensated for taking on the additional production risk through higher premiums.

The analytic results were confirmed using a numerical example. The processor's expected profits under bushel contracts were shown to be greater than when acreage contracts were used. Premium levels and the number of farmers under contract were also greater under bushel contracts, implying greater expected producer profits compared to the case of acreage contracts. Furthermore, bushel contracts resulted in relatively greater increases in the processor's expected profits as the spatial correlation of yields increased. Thus, bushel contracts may be more prevalent when yields within the processor's contracting regions are highly correlated. While the model is highly stylized, the basic framework provides for a host of possible extensions.

In Chapter 3, focus shifted to the production literature and the relationship between inputs and production risk. The goal of Chapter 3 was to identify and reconcile an apparent paradox in the academic literature regarding risk preferences and the optimal use of fertilizer. Empirical evidence implies that production variability is increasing in the amount of fertilizer applied. Therefore, a risk-averse farmer will use less fertilizer than a risk-neutral farmer. However, empirical evidence also implies that farmers consistently over-apply fertilizer. This is explained as a form of self-protection and a reaction to input uncertainty caused by stochastic nutrient gains and losses in the soil. The paradox of these seemingly opposing views is what is defined as the fertilizer problem.

Using experimental yield response data, stochastic relationships between yields and available soil nutrients, and soil nutrients and applied fertilizer were estimated. The estimation results support previous empirical evidence that yield variability increases with fertilizer application rates. Optimal application rates were then computed numerically for a risk-averse and risk-neutral farmer. The risk-averse farmer does indeed use less fertilizer than the risk-neutral farmer. However, both risk-neutral and risk-averse farmers may optimally over-apply fertilizer in response to input uncertainty.

In addition to the empirical analysis of the experimental data, the results from a farmer survey were also presented to attempt to gain additional insight into producer's optimal input decisions. The results imply that farmers prefer "risk" when it involves realizations above the mean compared to realizations below the mean, and that farmers prefer certain outcomes to gambles. Moreover, while roughly half of the respondents felt that yield variability increases with fertilizer use, less than 30% felt that increased fertilizer use increased yield risk. The survey results tend to support claims that farmers may over-apply fertilizer as an act of self-protection in that they are more averse to the risk of low yields or losses than they are to the risk of high yield realizations. However, a shortcoming of the globally concave utility function and expected utility theory is that the variability of uncertain prospects is discounted equally regardless of whether the risk is above or below the average. The survey results imply that this is not how farmers formulate preferences over potential yield gambles.

In Chapter 5, the focus was turned to weather variability. Specifically, a methodological framework was presented for developing an insurance program based on weather. The method outlined the estimation of unbiased rainfall histories when historical

data may be sparse through the use of spatial kriging and Markov Chain Monte Carlo methods. An application of the methodology was provided using a rich data set of historical rainfall in Iowa. The method was shown to provide unbiased rainfall histories which can be used to rate an actuarially sound insurance program based on rainfall. An example drought insurance contract was also outlined. The actuarially fair premium rates were calculated using Monte Carlo analysis. Finally, a historical analysis was provided to validate the effectiveness of the methodology. The methodological framework could easily be generalized for applications in other regions and for other crops. The creation of viable weather-based insurance programs in developing regions is just one example of a potential application of the methodology.